

Risk Oriented Earthquake Hazard Assessment

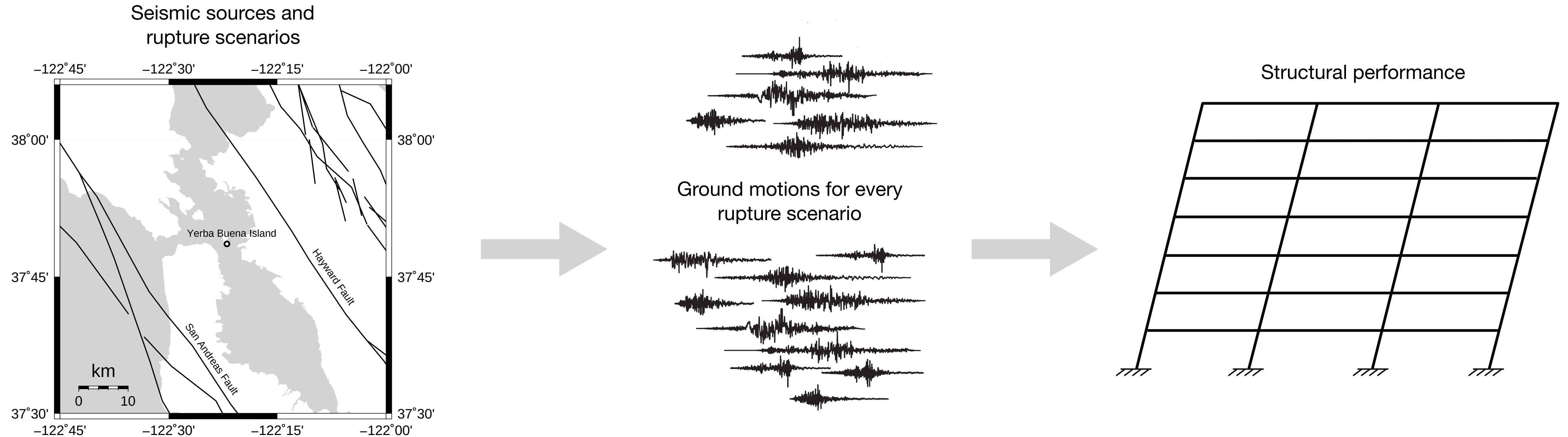
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Outline

- Seismic hazard within risk applications
- Requirements of ground-motion and correlation models
- Implications of the ergodic assumption for seismic risk
- Non-ergodic issues within seismic sequences

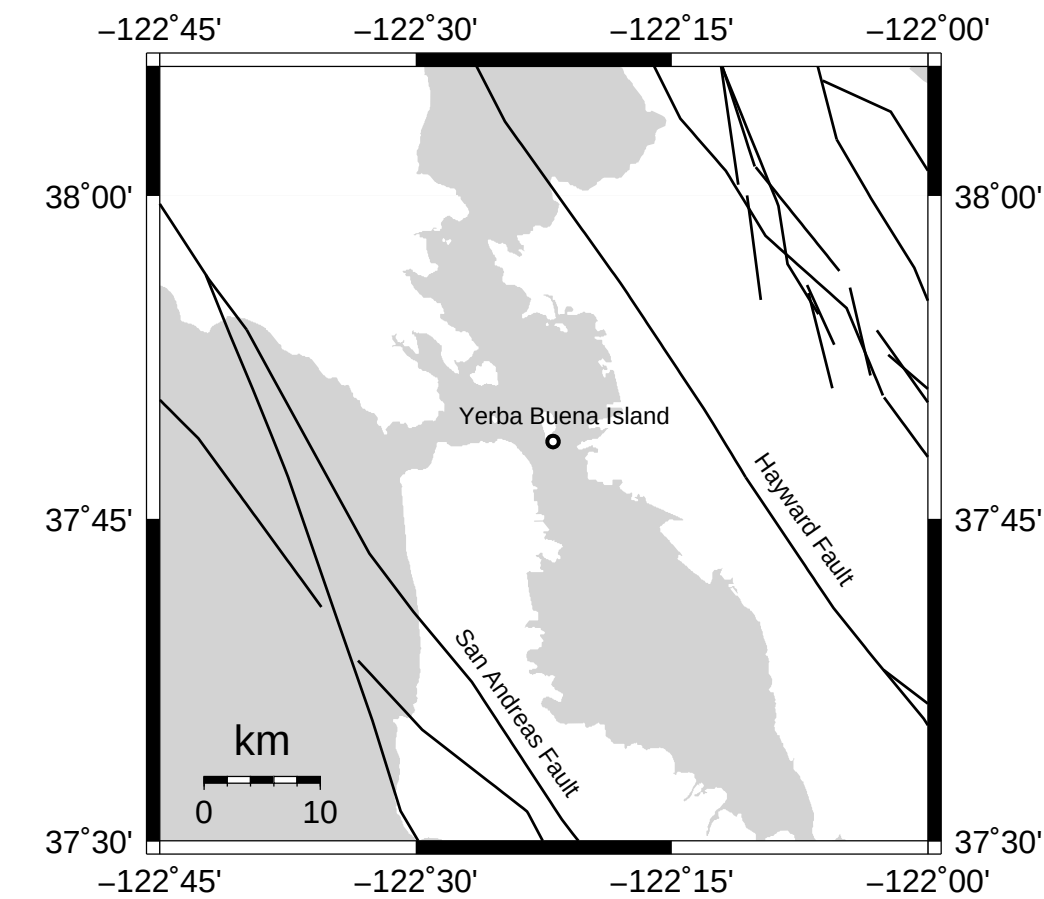
Conceptual risk framework



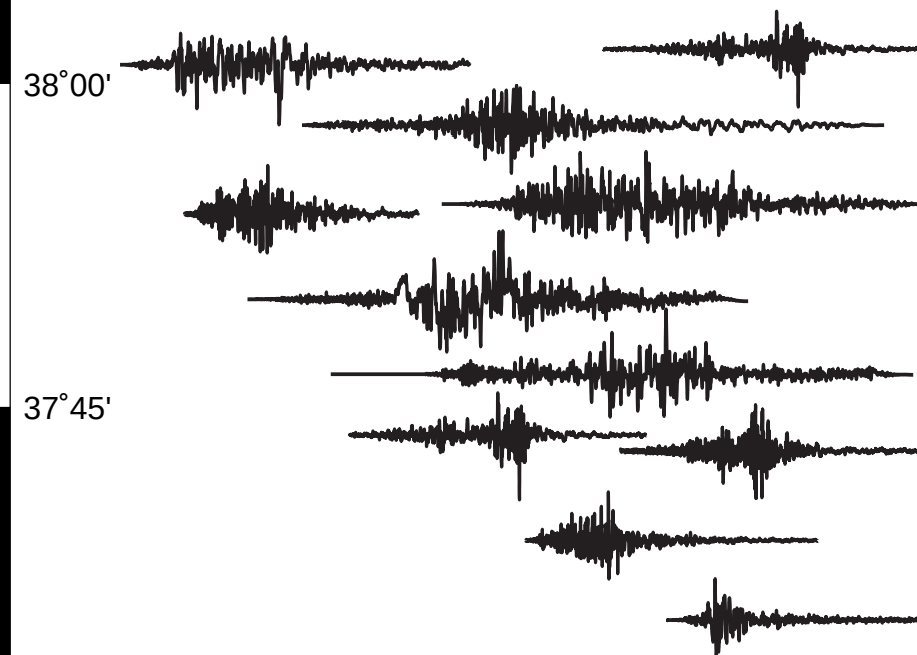
- From a seismic source model we define $\lambda(rup)$, the $P(rup)$ in some time window, or some stochastic sequence of rup scenarios
- For each rup we could pass compatible ground motions directly through the structural model and obtain the response distribution $f_{EDP}(edp; rup)$
- Loss estimates are derived from the conditional distribution of $EDP \mid rup$

Typical risk framework (single site)

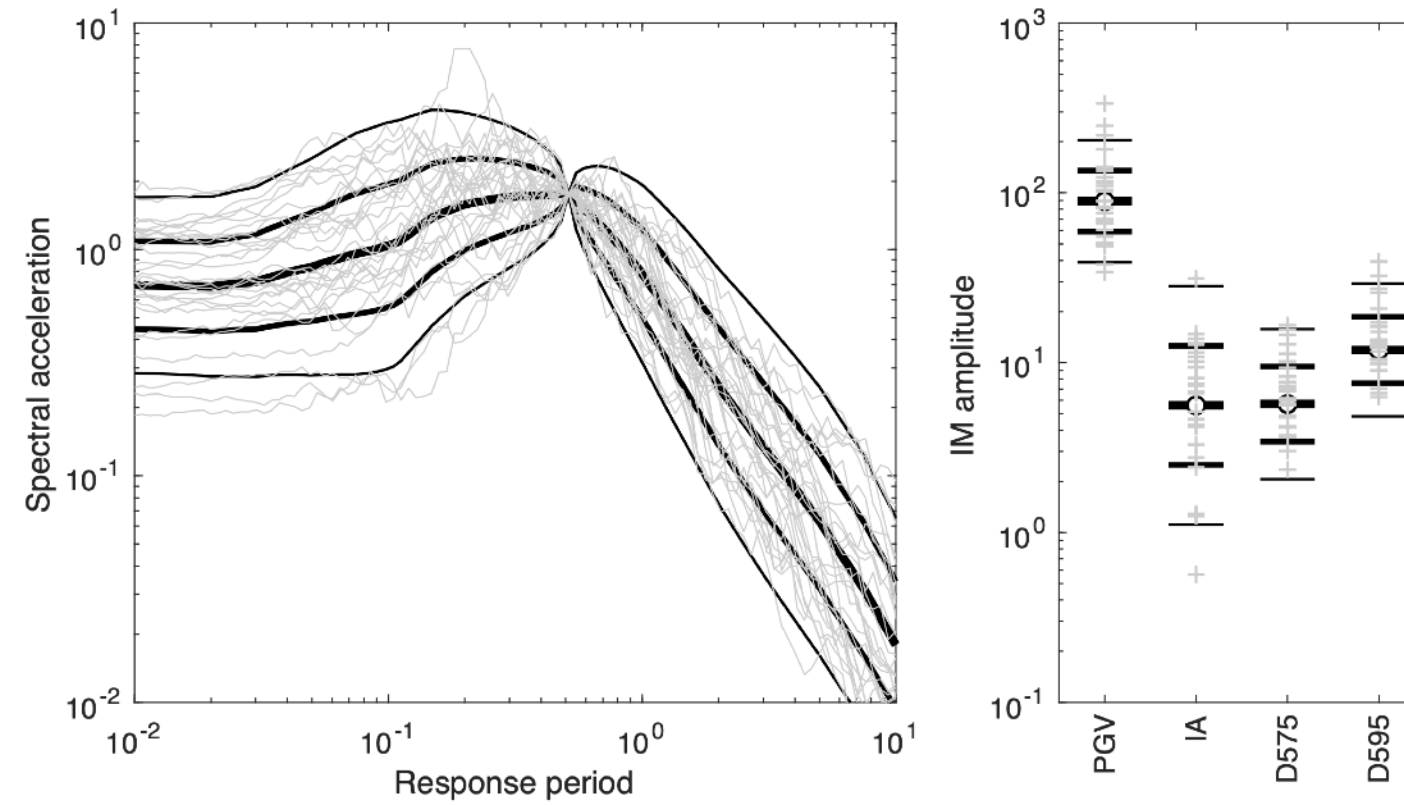
Seismic sources and rupture scenarios



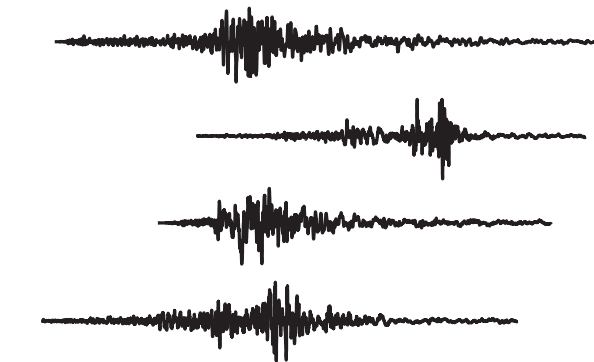
Ground motions used in GMM development



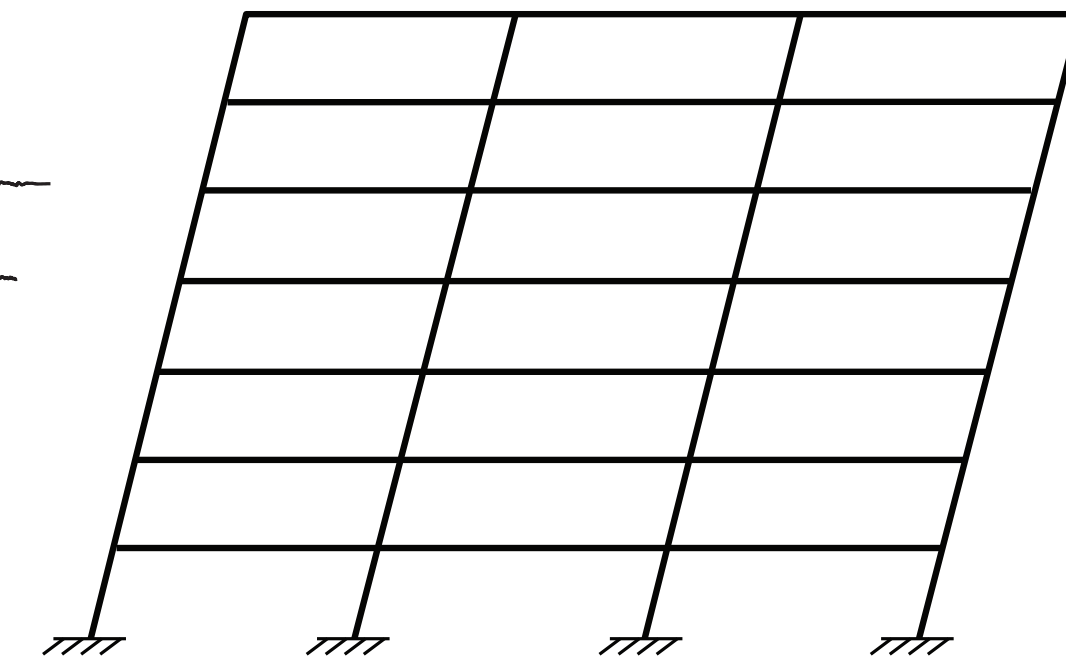
Target intensity measures



Selected ground motions

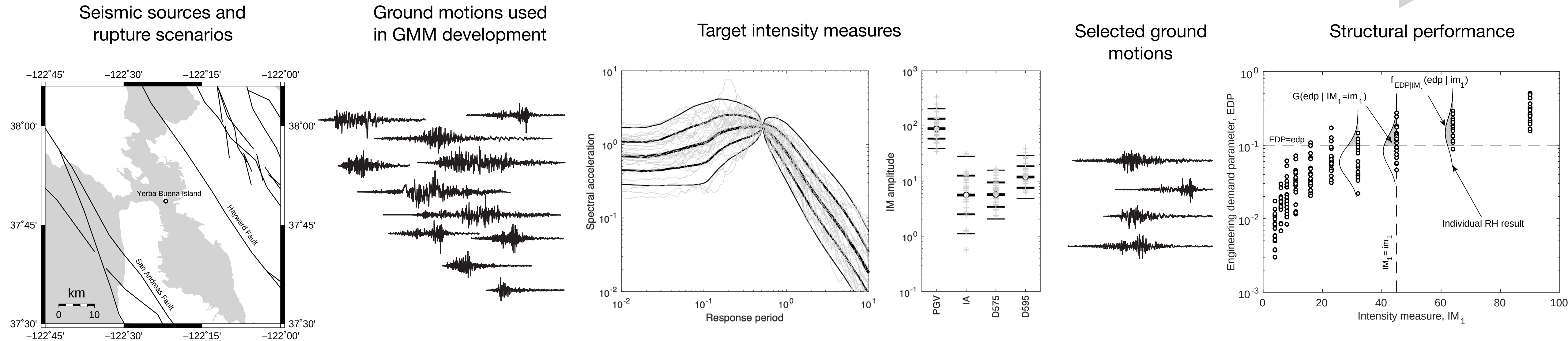


Structural performance



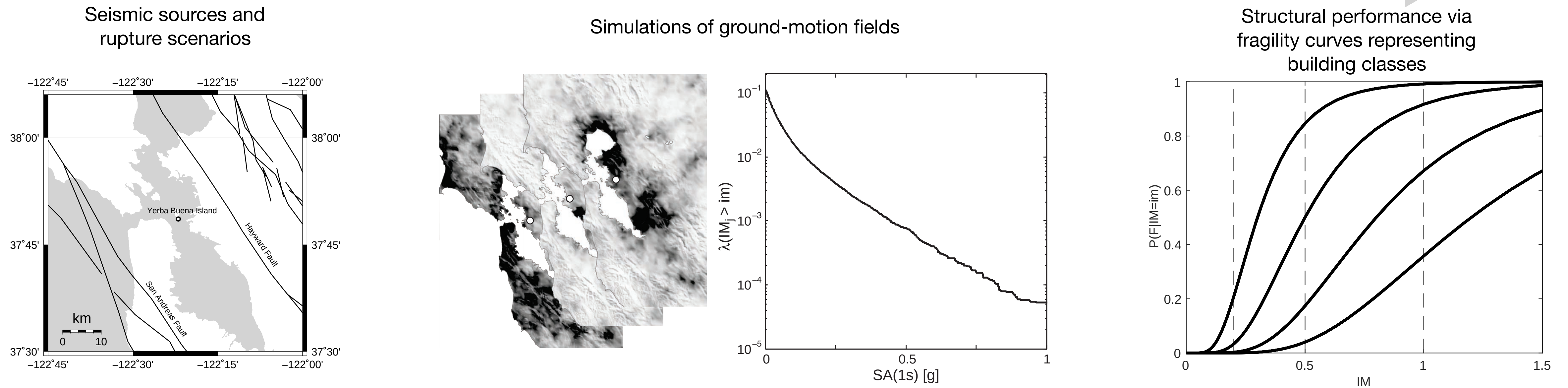
- A seismic source model defines $\lambda(rup)$, the $P(rup)$ in a time window, or some stochastic sequence of rup scenarios
- Ground-motion models and correlation models are used to predict the distribution of intensity measures $f_{IM}(\mathbf{im}; rup)$ for each rupture
- A relatively small suite of ground motions are selected to be representative of $f_{IM}(\mathbf{im}; rup)$, and upon passing these through the structural model we estimate $f_{EDP|IM}(\mathbf{edp} | \mathbf{im}; rup)$
- Loss estimates are derived from the conditional distribution of $\mathbf{EDP} | \mathbf{IM}$

Typical risk framework (single site)



- A seismic source model defines $\lambda(rup)$, the $P(rup)$ in a time window, or some stochastic sequence of rup scenarios
- Ground-motion models and correlation models are used to predict the distribution of a single intensity measure $f_{IM_1}(im_1; rup)$ for each rupture
- A relatively small suite of ground motions are selected to be representative of $f_{IM|IM_1}(im | im_1; rup)$, and upon passing these through the structural model we estimate $f_{EDP|IM_1}(edp | im_1; rup)$
- Loss estimates are derived from the conditional distribution of $EDP | IM_1$

Typical risk framework (portfolio)



- From a seismic source model we define $\lambda(rup)$, the $P(rup)$ in some time window, or some stochastic sequence of rup scenarios
- Ground-motion models and correlation models are used to simulate fields of intensity measures according to $\mathbf{im} \sim MVN [(\boldsymbol{\mu}(rup), \boldsymbol{\Sigma}(rup)]$
- Loss estimates are derived from fragility curves $\mathbf{DM} | IM$

Typical risk framework

- ◉ Our objective is to calibrate these various different approaches to solving the same problem such that they provide similar results
- ◉ For portfolio risk applications it is clear that simplifications must be made in light of the computational and data requirements
- ◉ However, it is still important that we attempt to ensure that the various characteristics of the ground-motion fields and structural portfolio are represented in our simplified methods
- ◉ The focus of this presentation is to discuss the requirements for ground-motion models used within portfolio risk applications and to highlight potential inconsistencies that exist in current approaches

Requirements for ground-motion models

Model assumptions

- The focus of the ground-motion model is solely upon $f(\mathbf{IM}; rup)$, but it is important ensure that the $\mathbf{IM}$ s that our model generates are consistent with what is used for the fragility modelling
- We assume that the distribution $f(\mathbf{IM}; rup)$ is well-represented by a multivariate normal distribution (for logarithmic intensity measures)

$$f(\mathbf{IM}; rup) \equiv MVN [\boldsymbol{\mu}(rup), \boldsymbol{\Sigma}(rup)]$$

- We therefore require:
 - $\boldsymbol{\mu}(rup)$ - which is provided by the ground motion model; and
 - $\boldsymbol{\Sigma}(rup)$ - which is the covariance matrix of the intensity measures for each rupture
- The current presentation focus upon $\boldsymbol{\Sigma}(rup)$, its constituent components, and its uncertainty

Traditional components of ground motion variability

General representation of variability components in ground-motion models for portfolio risk applications

Simulated surface
motion (linear case)

Between-event
variate

$$im(\mathbf{x}) = \mu(\mathbf{x}; rup) + \delta_B + \delta_W(\mathbf{x})$$

Median surface
prediction (linear case)

Within-event
variate

Nonlinear site response complicates this a little as we first predict an motion at some reference velocity horizon (at depth), and then compute the amplification as a function of this motion

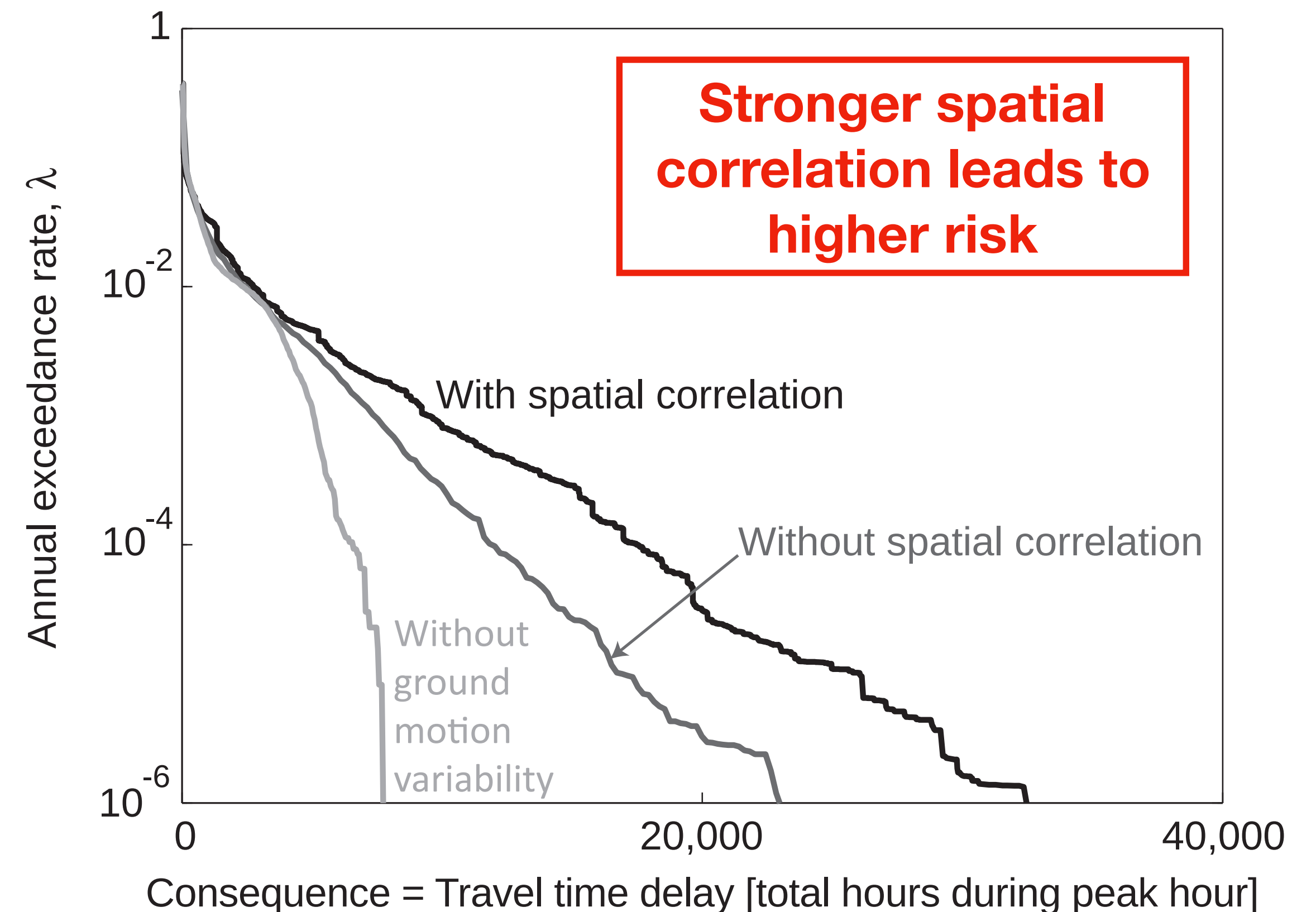
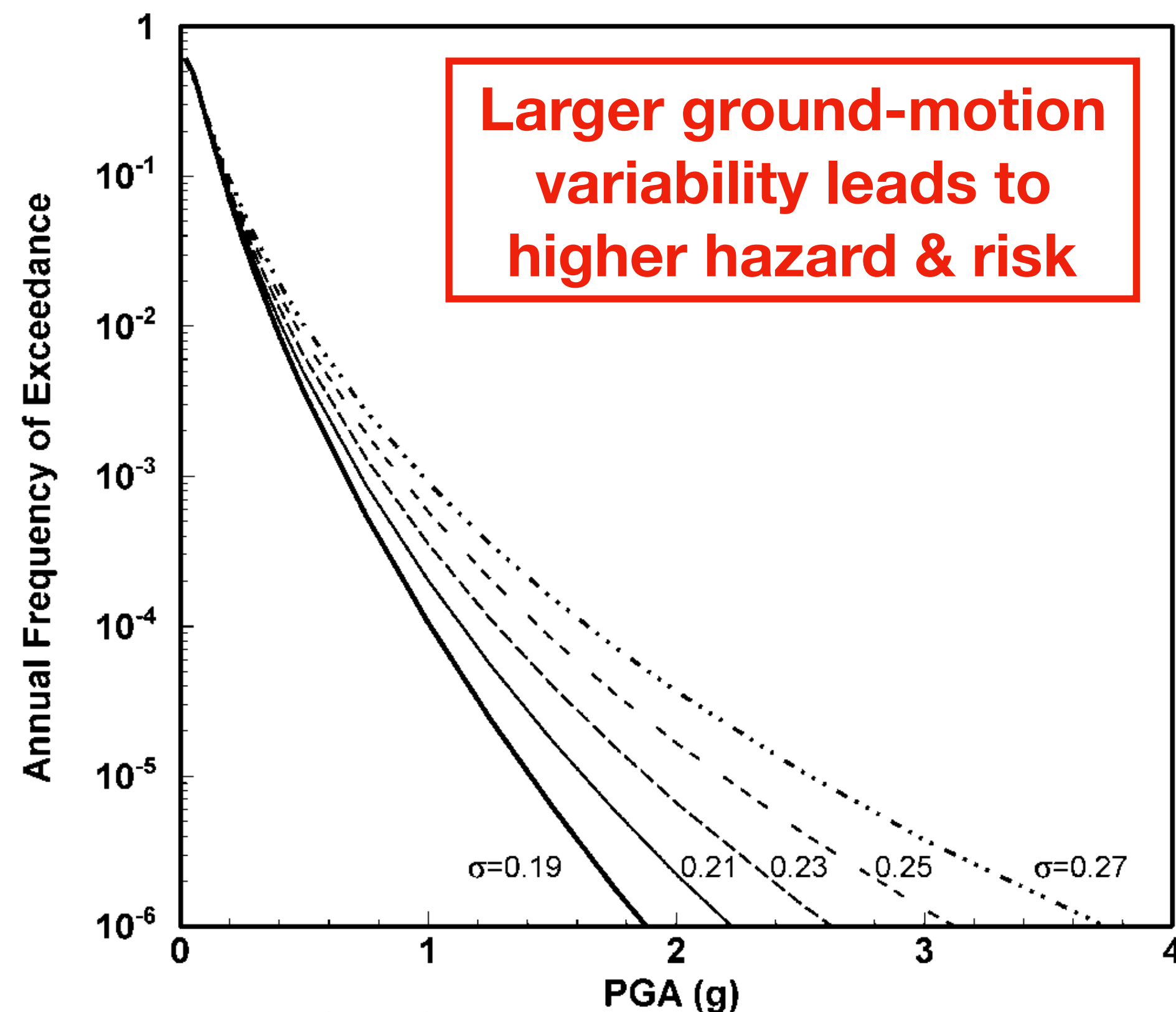
Motion at depth: $im_{ref}(\mathbf{x}) = \mu_{ref}(\mathbf{x}; rup) + \delta_{B,0} + \delta_{W,0}(\mathbf{x})$

Motion at surface: $im(\mathbf{x}) = im_{ref}(\mathbf{x}) + \mu_{\ln AF} \left[\mathbf{x}, im_{ref}(\mathbf{x}) \right] + \varepsilon_{\ln AF} \left[\mathbf{x}; im_{ref}(\mathbf{x}) \right]$

Traditional variance structure

Event variates are normal with variance τ^2 , within-event variates are multi-variate normal with marginal variance ϕ^2 and a spatial correlation matrix $\mathbf{R}(\mathbf{x})$

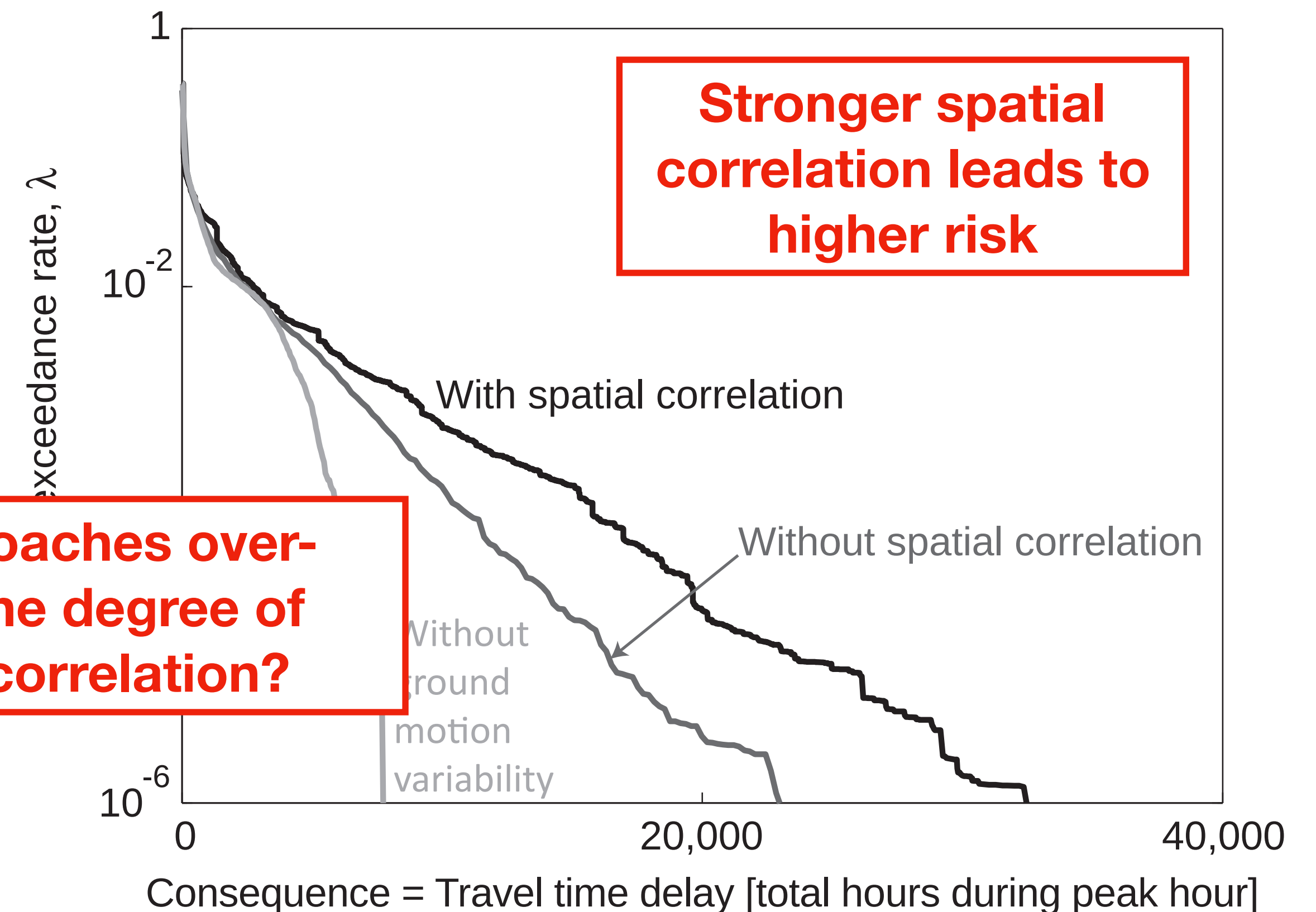
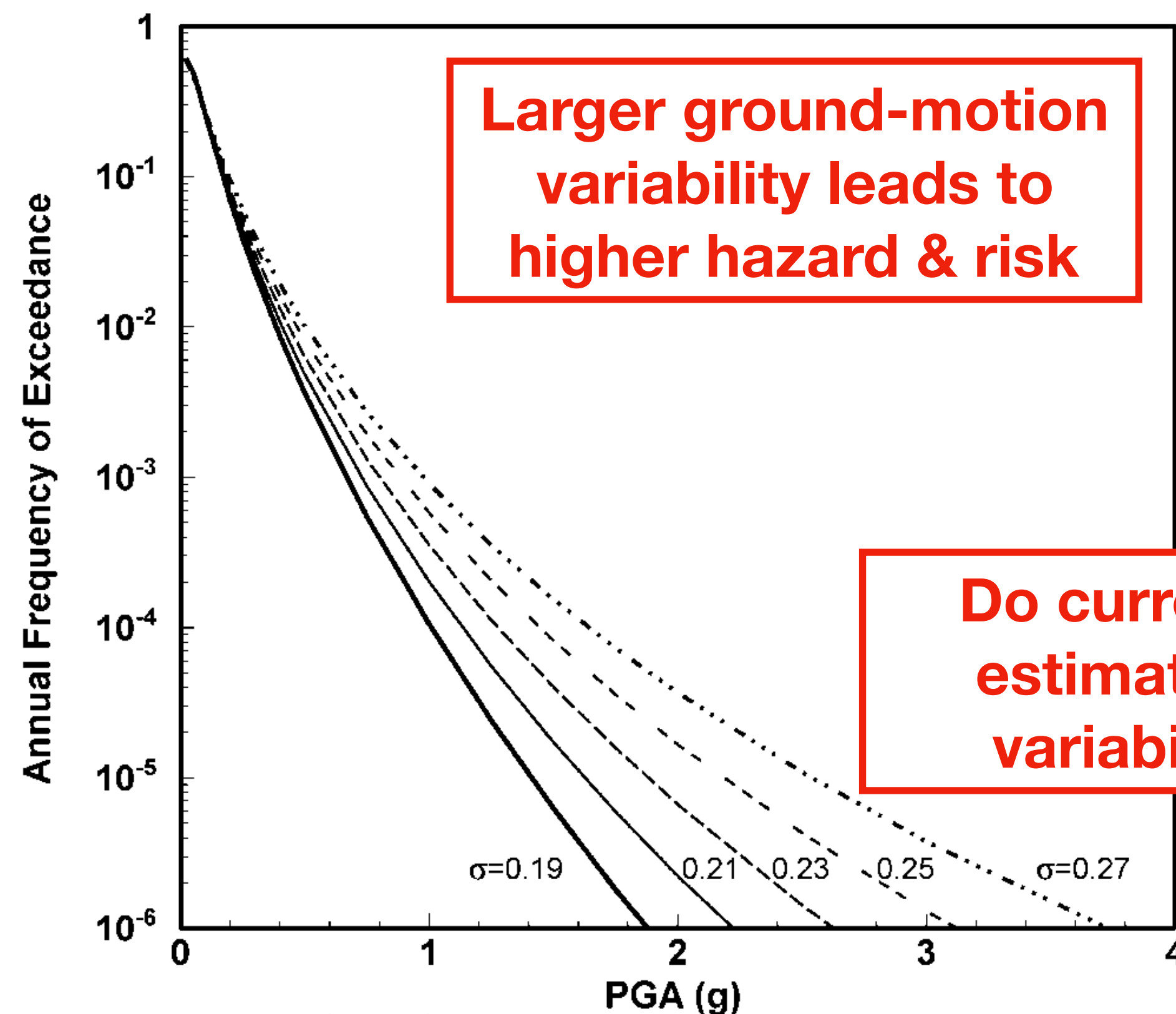
$$im(\mathbf{x}) = \mu(\mathbf{x}; rup) + \delta_B + \delta_W(\mathbf{x}) \quad \delta_B \sim N(0, \tau^2) \quad \delta_W(\mathbf{x}) \sim MVN[\mathbf{0}, \phi^2(\mathbf{x})\mathbf{R}(\mathbf{x})] \quad \sigma(\mathbf{x}) = \sqrt{\tau^2 + \phi^2(\mathbf{x})}$$



Traditional variance structure

Event variates are normal with variance τ^2 , within-event variates are multi-variate normal with marginal variance ϕ^2 and a spatial correlation matrix $\mathbf{R}(\mathbf{x})$

$$im(\mathbf{x}) = \mu(\mathbf{x}; rup) + \delta_B + \delta_W(\mathbf{x}) \quad \delta_B \sim N(0, \tau^2) \quad \delta_W(\mathbf{x}) \sim MVN[\mathbf{0}, \phi^2(\mathbf{x})\mathbf{R}(\mathbf{x})] \quad \sigma(\mathbf{x}) = \sqrt{\tau^2 + \phi^2(\mathbf{x})}$$



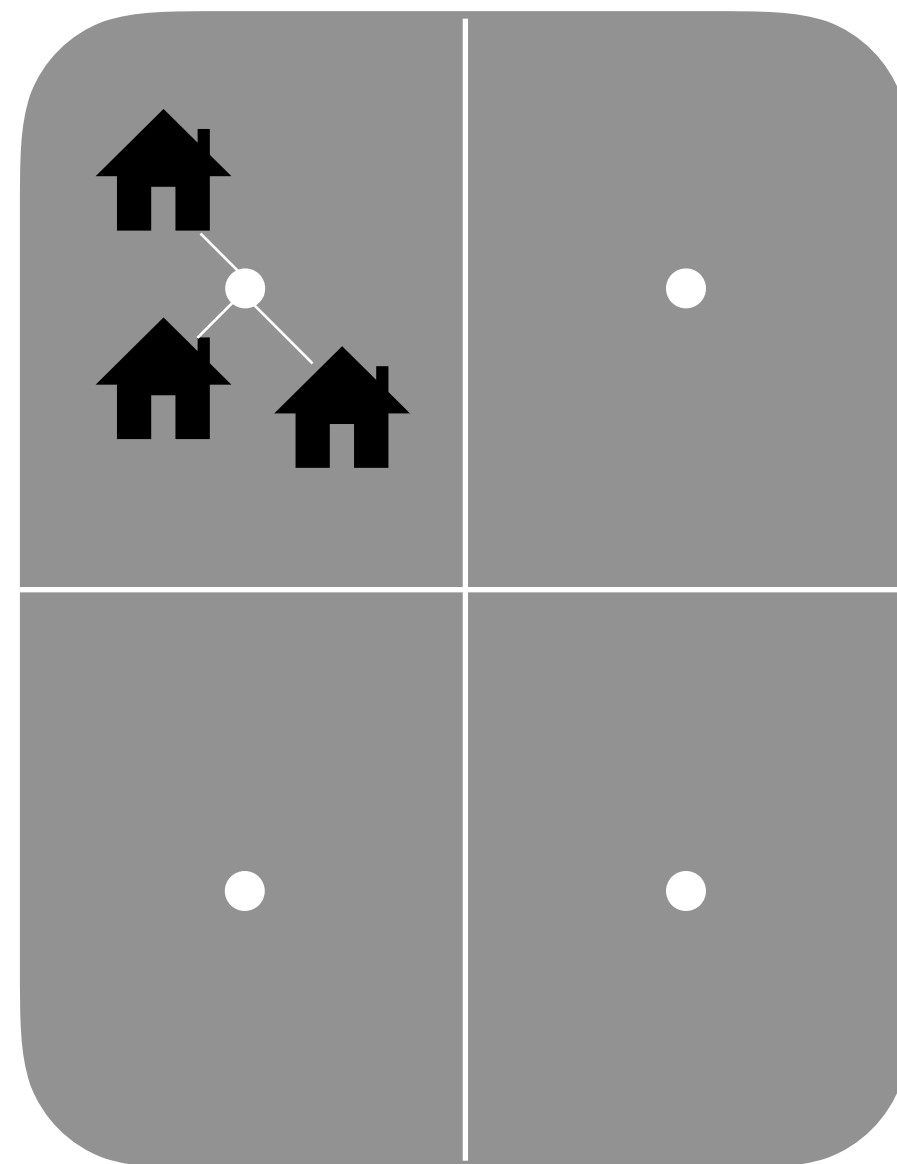
Do current approaches over-estimate both the degree of variability and correlation?

Correlation cases

For a general risk model there are various correlations among IMs that need to be considered

‘Same’
buildings,
‘same’ position

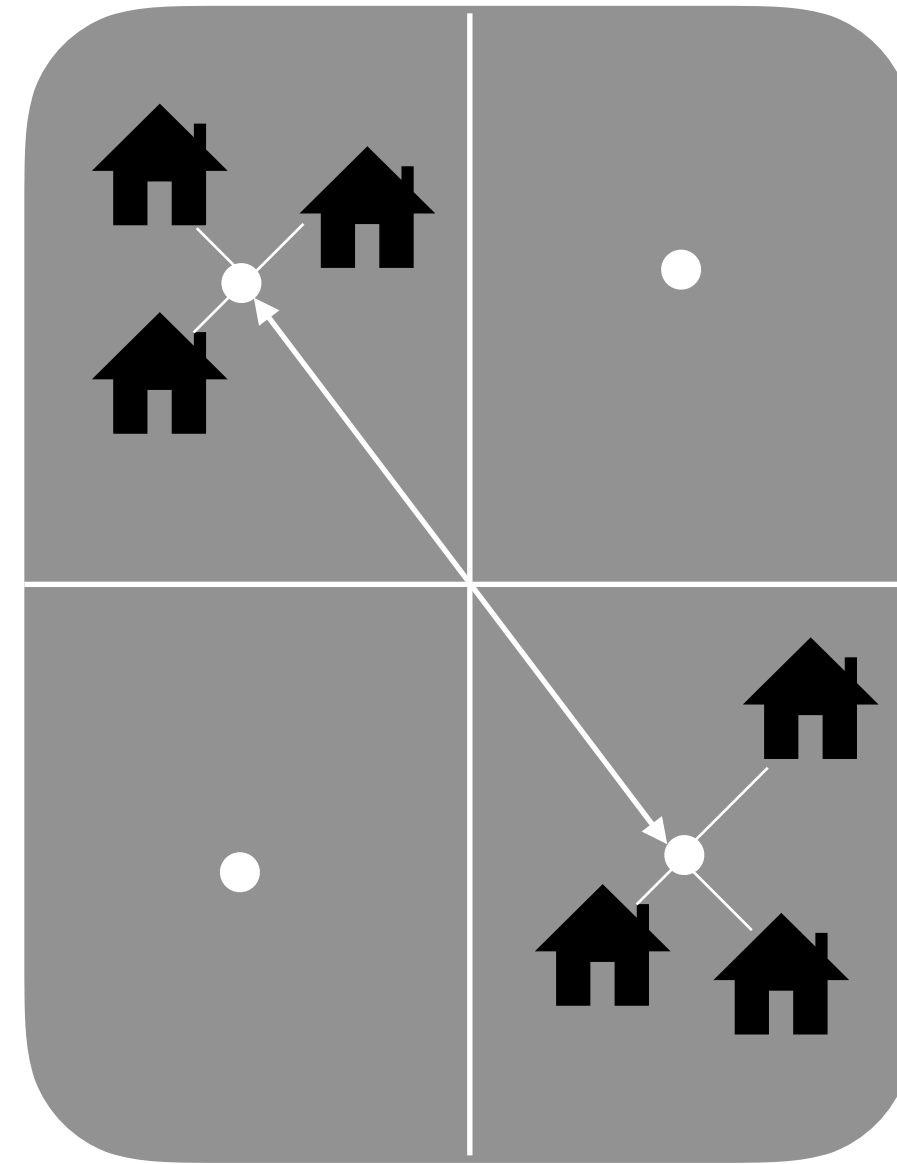
$$\rho(\{T_1, \mathbf{x}_1\}, \{T_1, \mathbf{x}_1\}) = 1$$



No correlation
required

‘Same’ buildings,
different positions

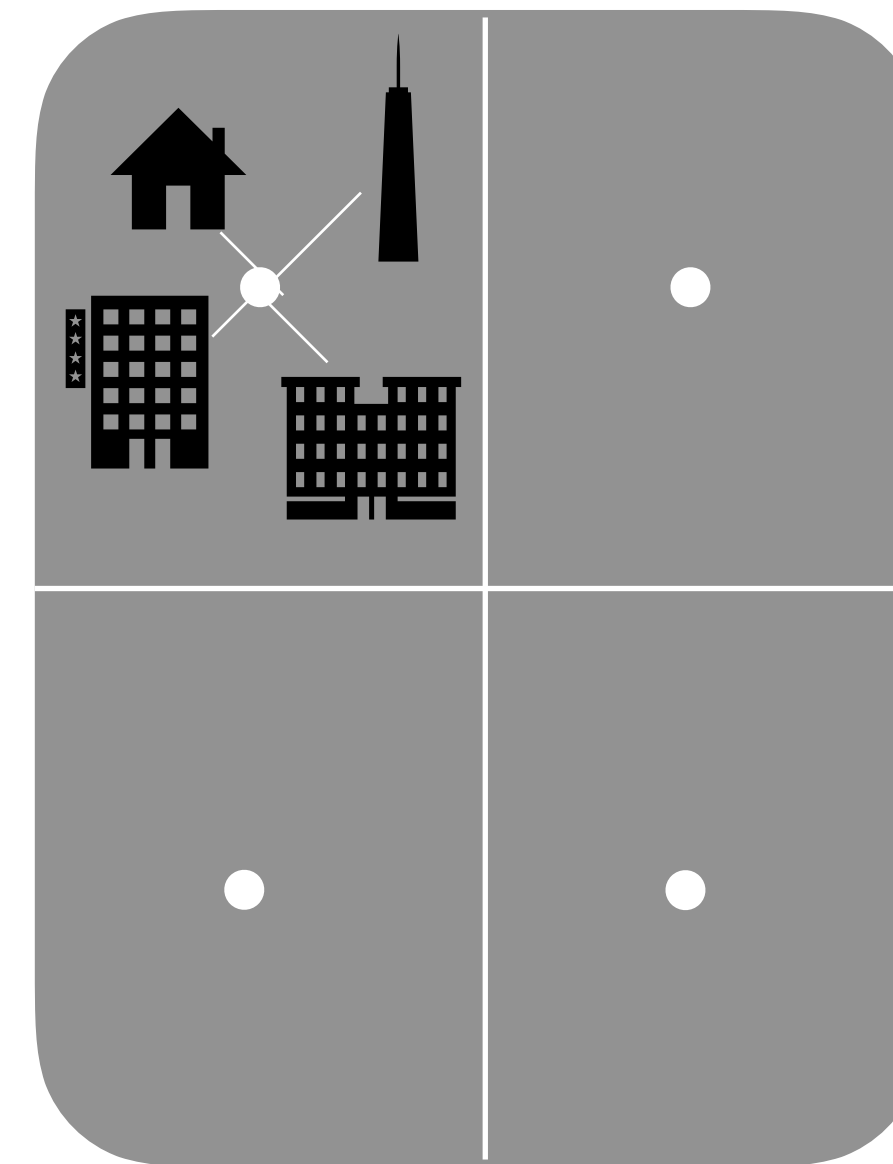
$$\rho(\{T_1, \mathbf{x}_1\}, \{T_1, \mathbf{x}_2\}) < 1$$



Only spatial
correlation

Different
buildings,
‘same’ position

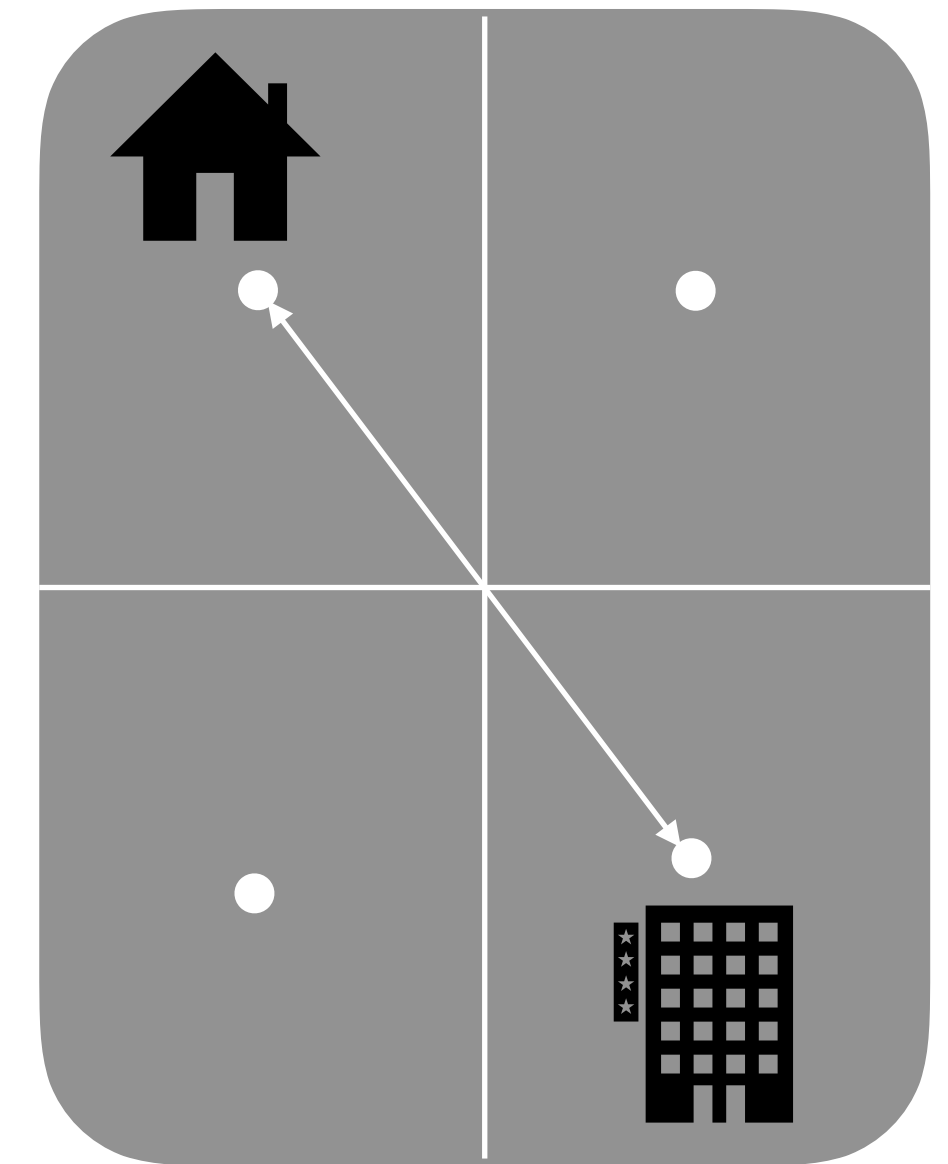
$$\rho(\{T_1, \mathbf{x}_1\}, \{T_2, \mathbf{x}_1\}) < 1$$



Only inter-IM
correlation

Different buildings,
different positions

$$\rho(\{T_1, \mathbf{x}_1\}, \{T_2, \mathbf{x}_2\}) < 1$$



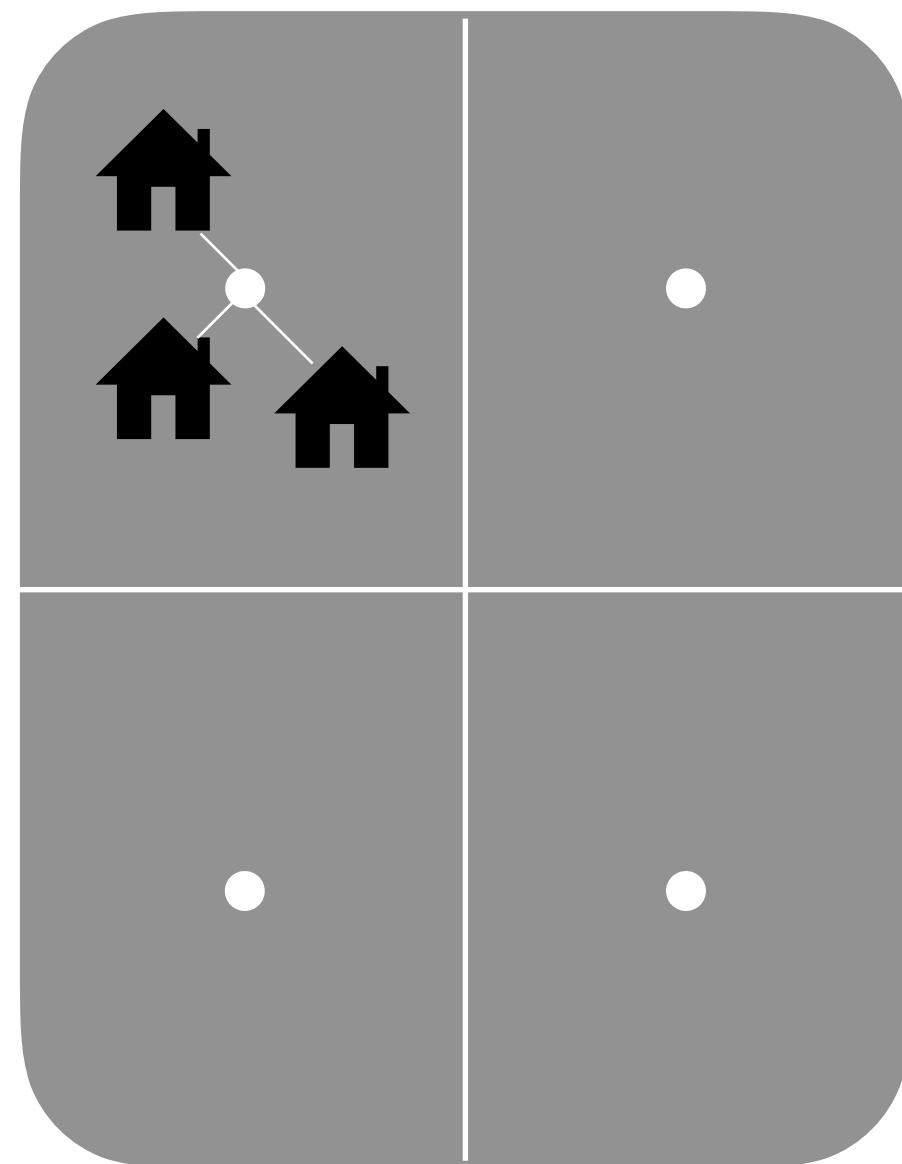
Spatial and inter-
IM correlation

Issues with treatment of correlation cases

For a general risk model there are various correlations among IMs that need to be considered

'Same'
buildings,
'same' position

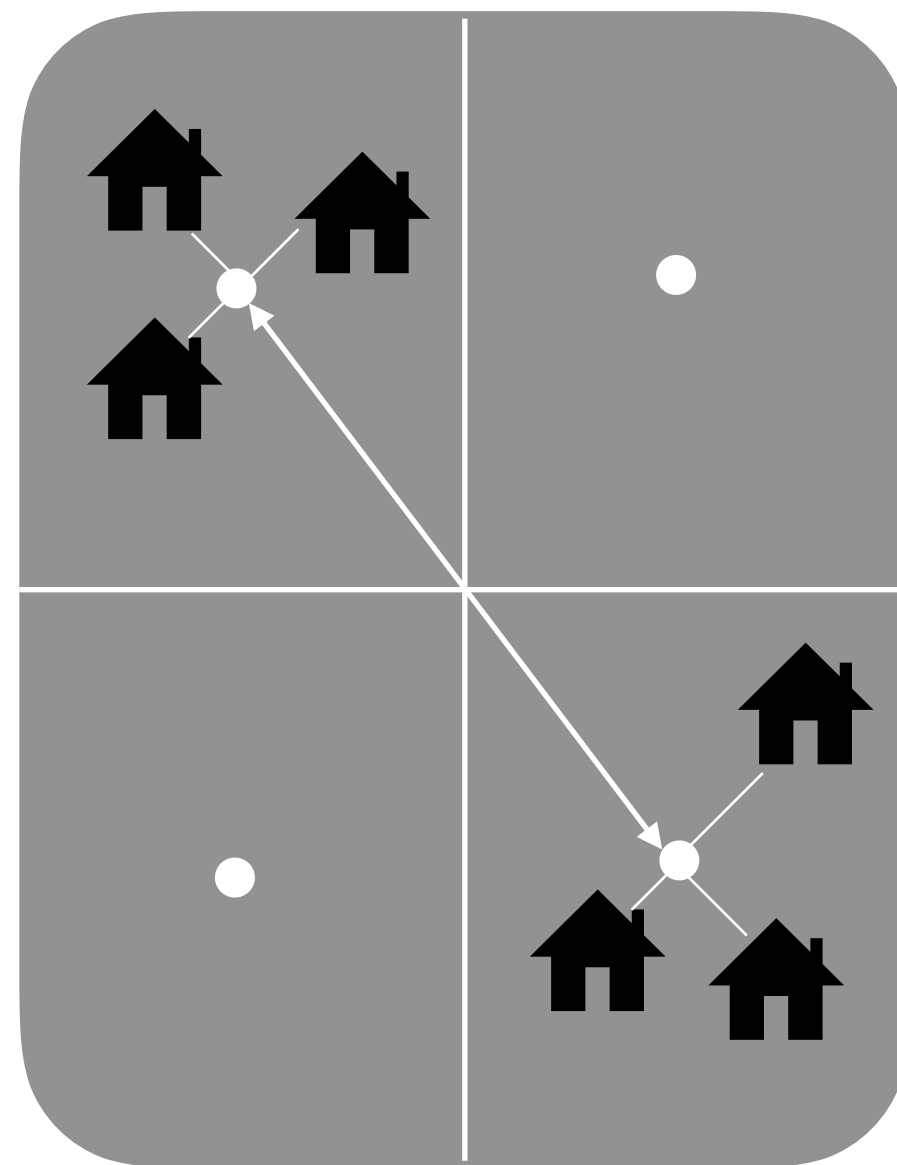
$$\rho(\{T_1, \mathbf{x}_1\}, \{T_1, \mathbf{x}_1\}) = 1$$



All buildings obviously not
at the same location -
cannot really have perfect
correlation within cell

'Same' buildings,
different positions

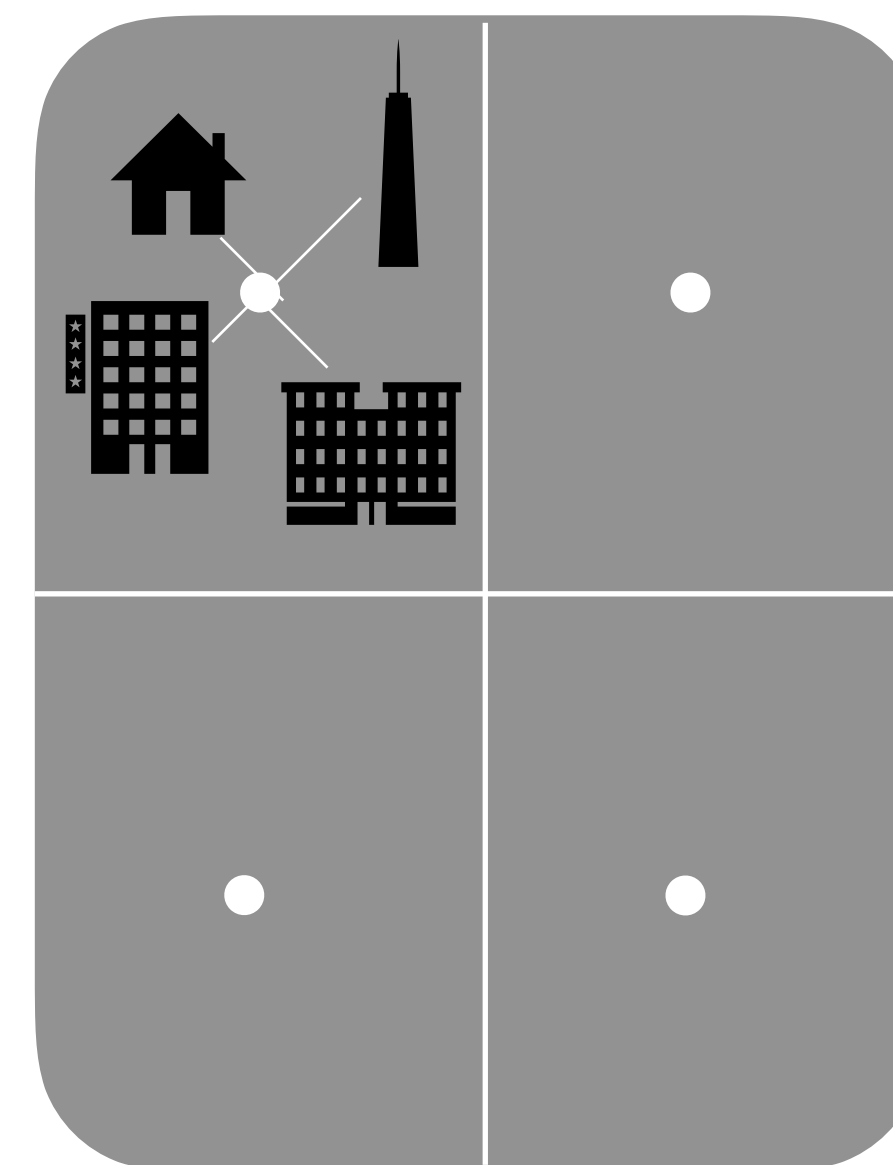
$$\rho(\{T_1, \mathbf{x}_1\}, \{T_1, \mathbf{x}_2\}) < 1$$



Pure spatial correlation
from point-to-point ignores
spatial distribution of
buildings within cells

Different
buildings,
'same' position

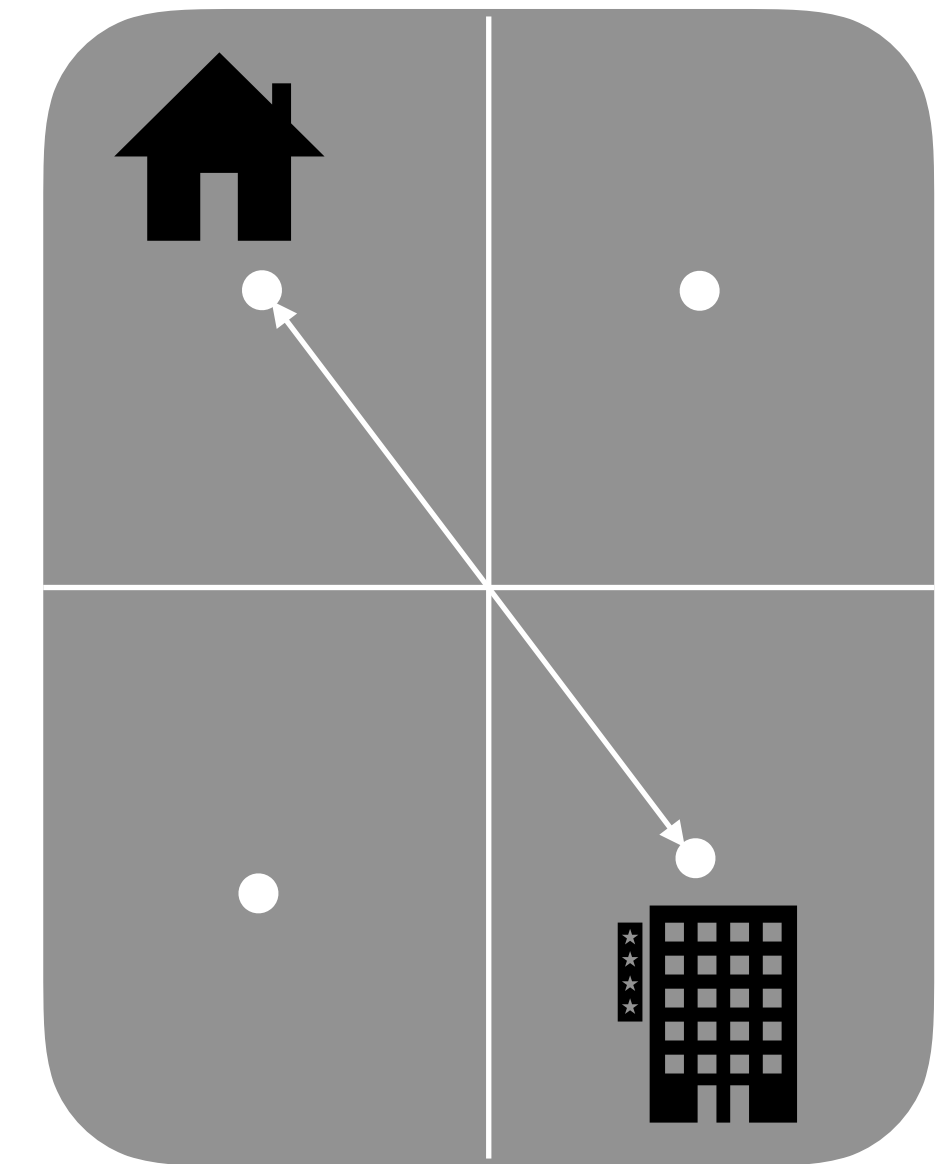
$$\rho(\{T_1, \mathbf{x}_1\}, \{T_2, \mathbf{x}_1\}) < 1$$



Need to down-weight the
typical inter-period
correlation to reflect spatial
distribution

Different buildings,
different positions

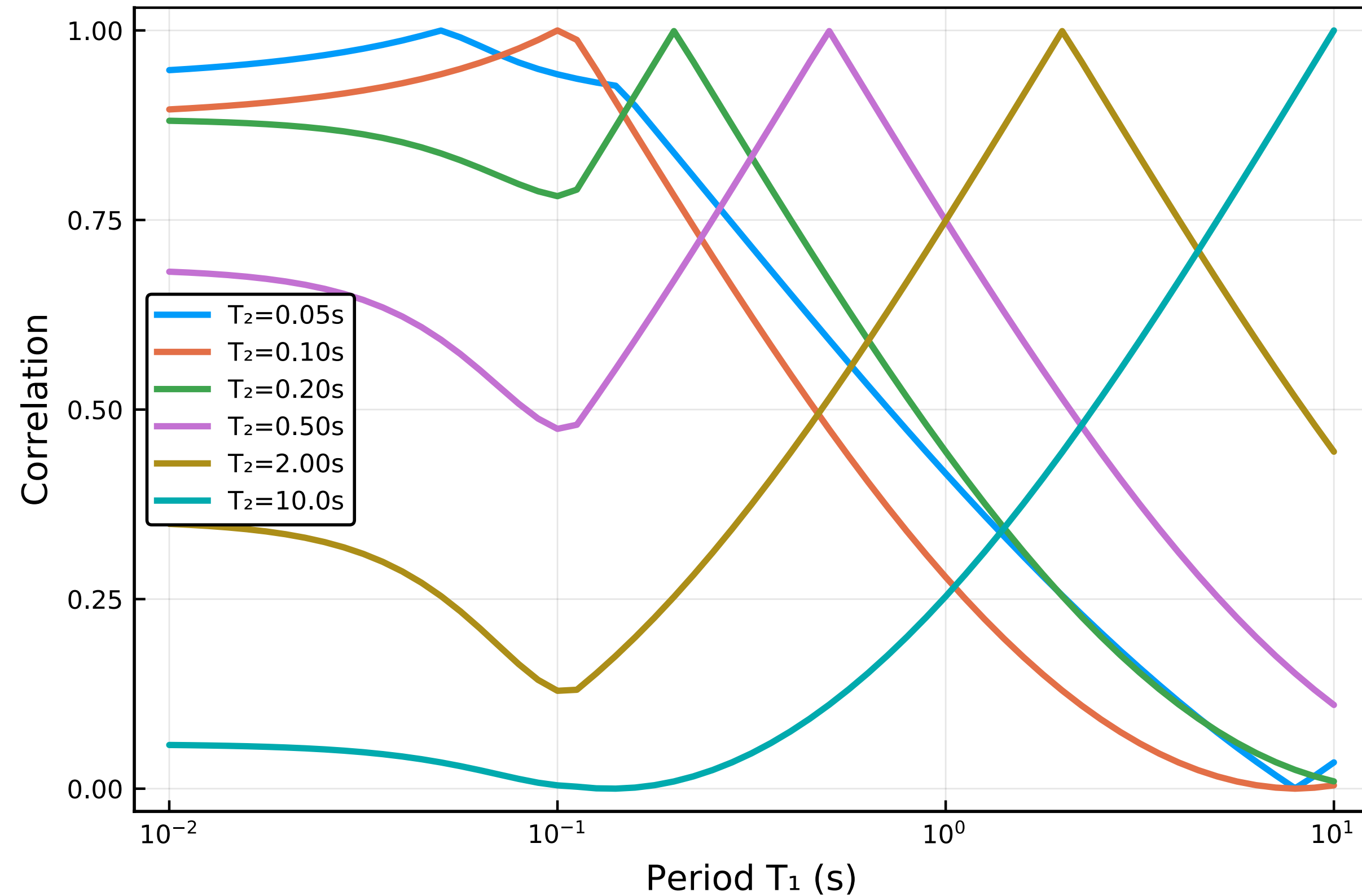
$$\rho(\{T_1, \mathbf{x}_1\}, \{T_2, \mathbf{x}_2\}) < 1$$



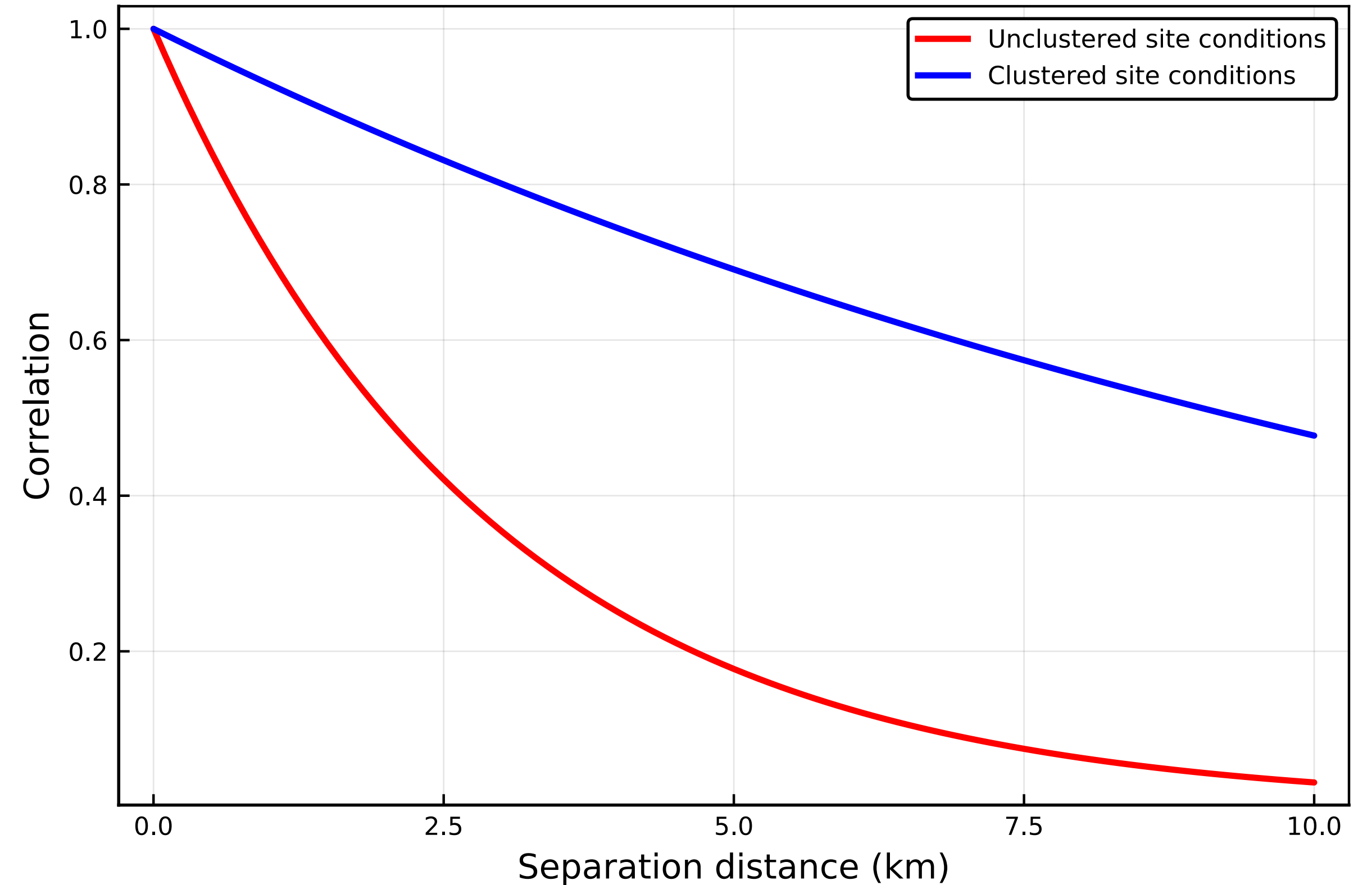
Have to consider possible
combinations of actual
locations for each building

Markovian approximation

Inter-period correlation from
Baker & Jayaram (2008), $\rho(T_1, T_2)$



Spatial correlation of
Jayaram & Baker (2009), $\rho(\mathbf{x}_1, \mathbf{x}_2 | \max(T_1, T_2))$



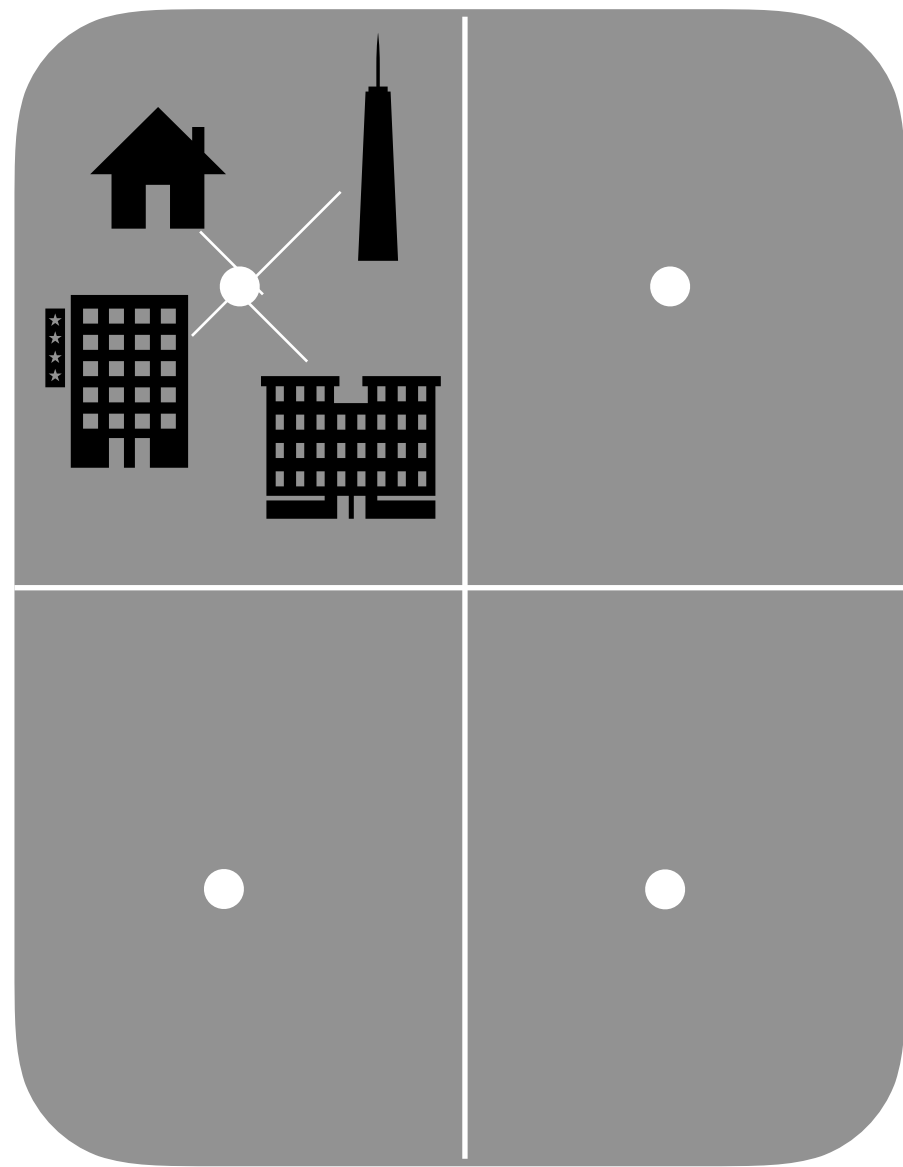
- The correlation between IM_p at location $\mathbf{x}_i$ and IM_q at location $\mathbf{x}_j$ can be very well approximated by the inter- IM correlation for zero separation distance, and the spatial correlation for the two locations based upon the lowest frequency IM

$$\rho\left(\{T_1, \mathbf{x}_1\}, \{T_2, \mathbf{x}_2\}\right) \approx \rho(T_1, T_2) \times \rho(\mathbf{x}_1, \mathbf{x}_2 | \max(T_1, T_2))$$

Effective inter-period correlations

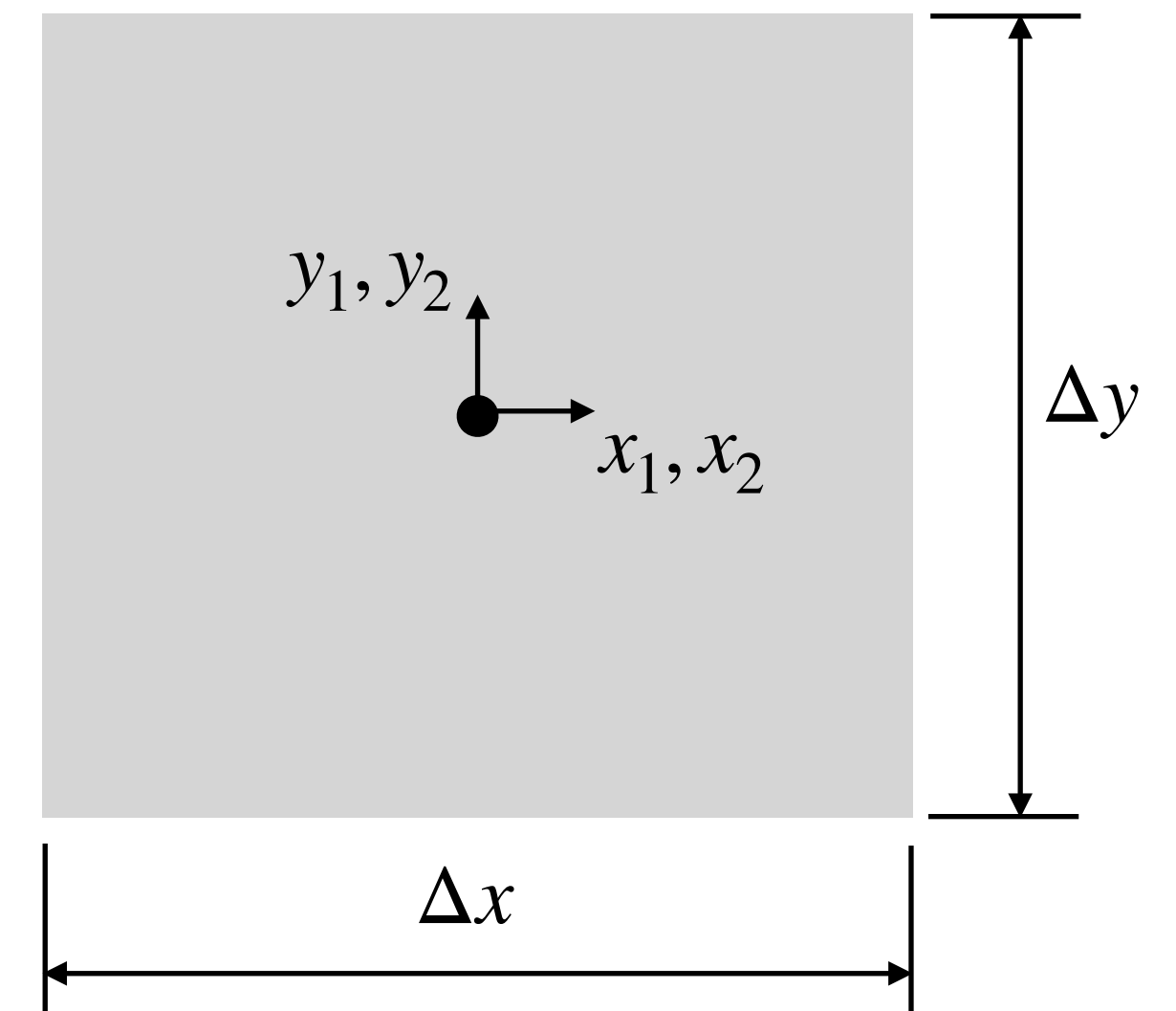
Different buildings,
'same' position

$$\rho(\{T_1, x_1\}, \{T_2, x_1\}) < 1$$



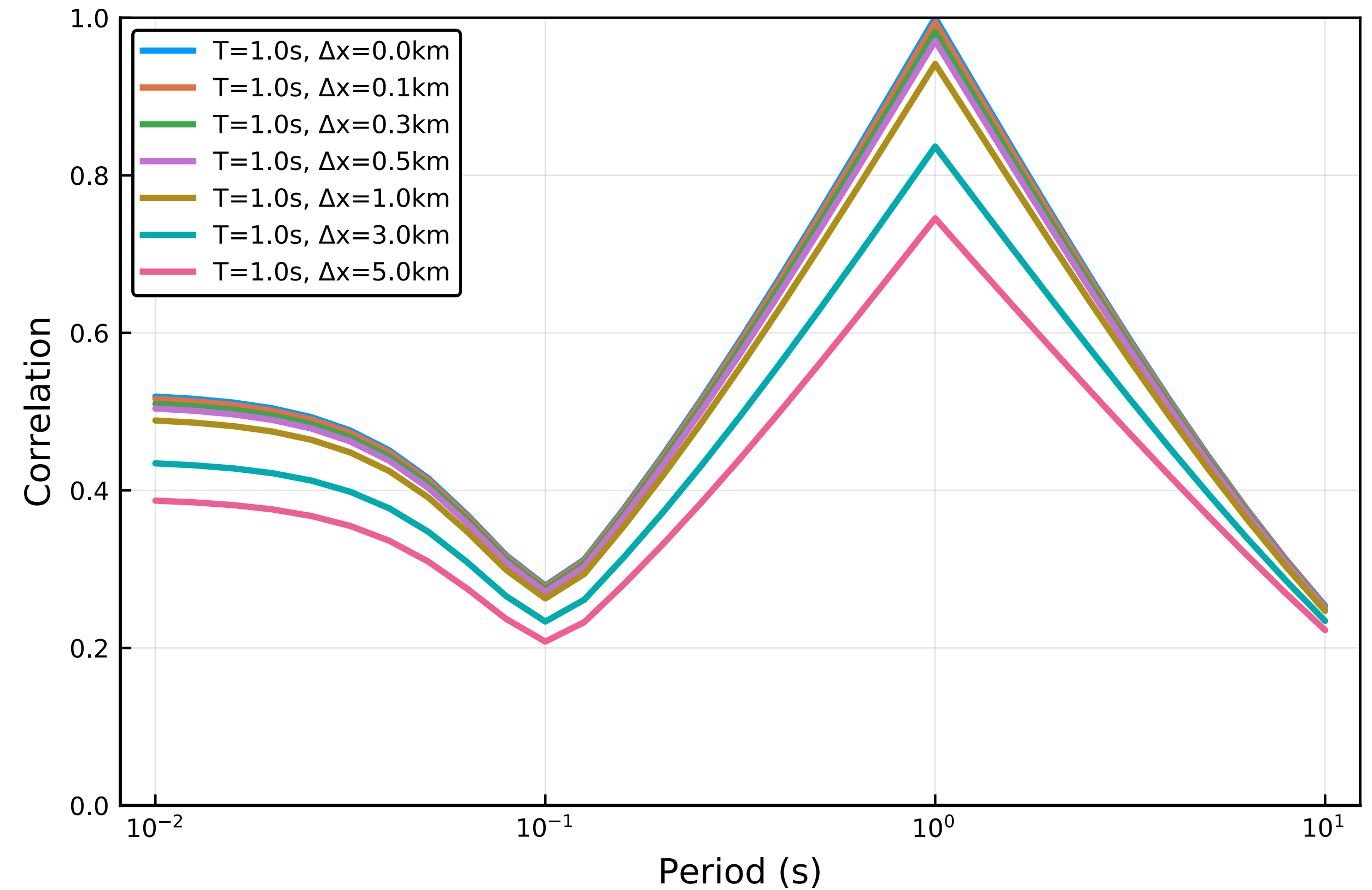
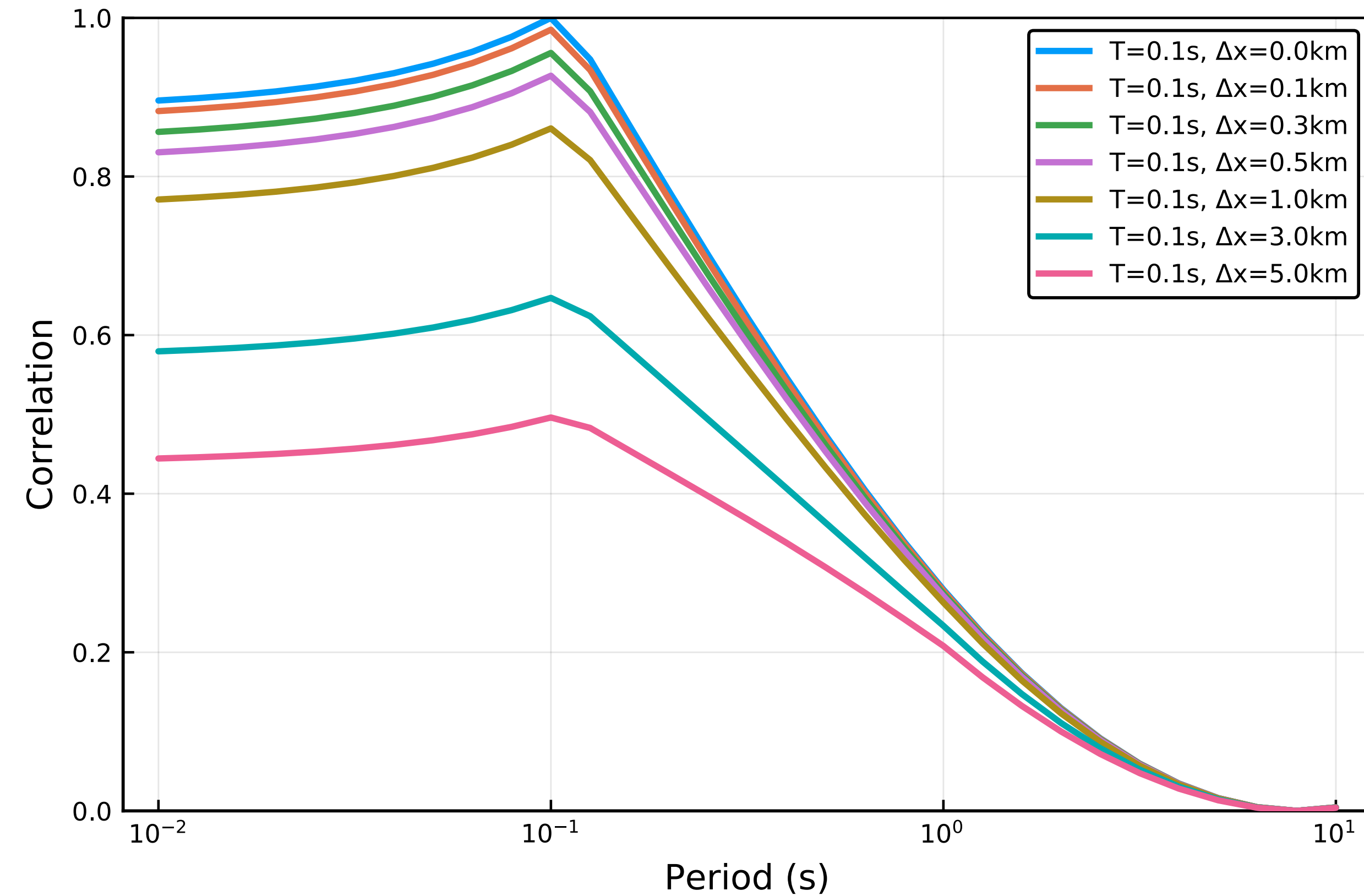
Need to down-weight the
typical inter-period
correlation to reflect spatial
distribution

- Correlation models that are normally used represent correlations among co-located *IMs*
- However, this isn't really what we want
- Even ignoring the fact that buildings within a class will have different periods (which should be accounted for in the fragility derivation), the spectral demands across buildings within a cell will vary due to their spatial separation



$$\rho_{eff}(T_1, T_2) = \frac{1}{\Delta x^2 \Delta y^2} \iiint \rho(\{x_1, y_1\}, \{x_2, y_2\}, T_1, T_2) dx_1 dx_2 dy_1 dy_2$$

Effective inter-period correlations



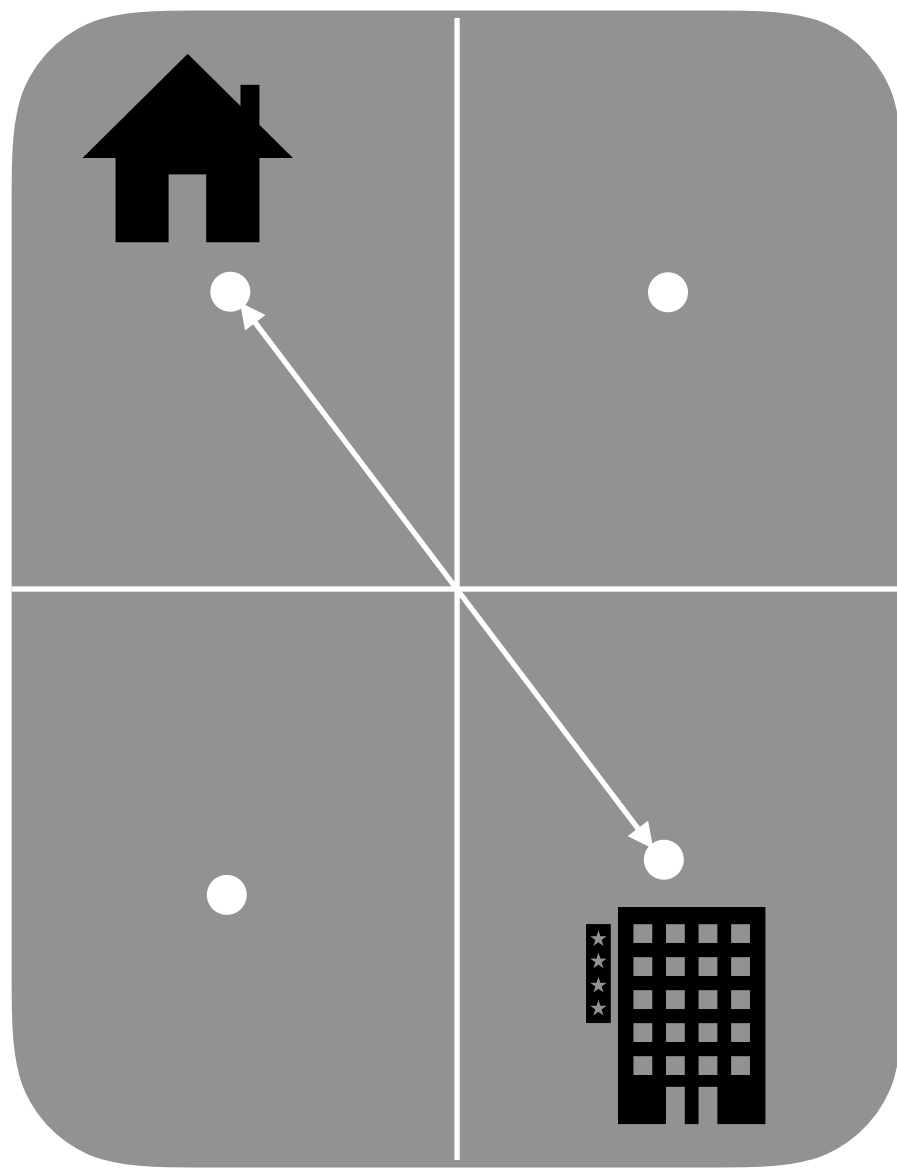
- Naturally, the larger the cell size, the greater the reduction in the inter-period correlation
- One needs to be working at a relatively high resolution in order to ignore this effect
- This reduced inter-period correlation also induces a slight increase in the ground-motion variability for the cell

$$\Delta\sigma(x) \approx \sigma(x)\sqrt{1 - \rho_{eff}^2}$$

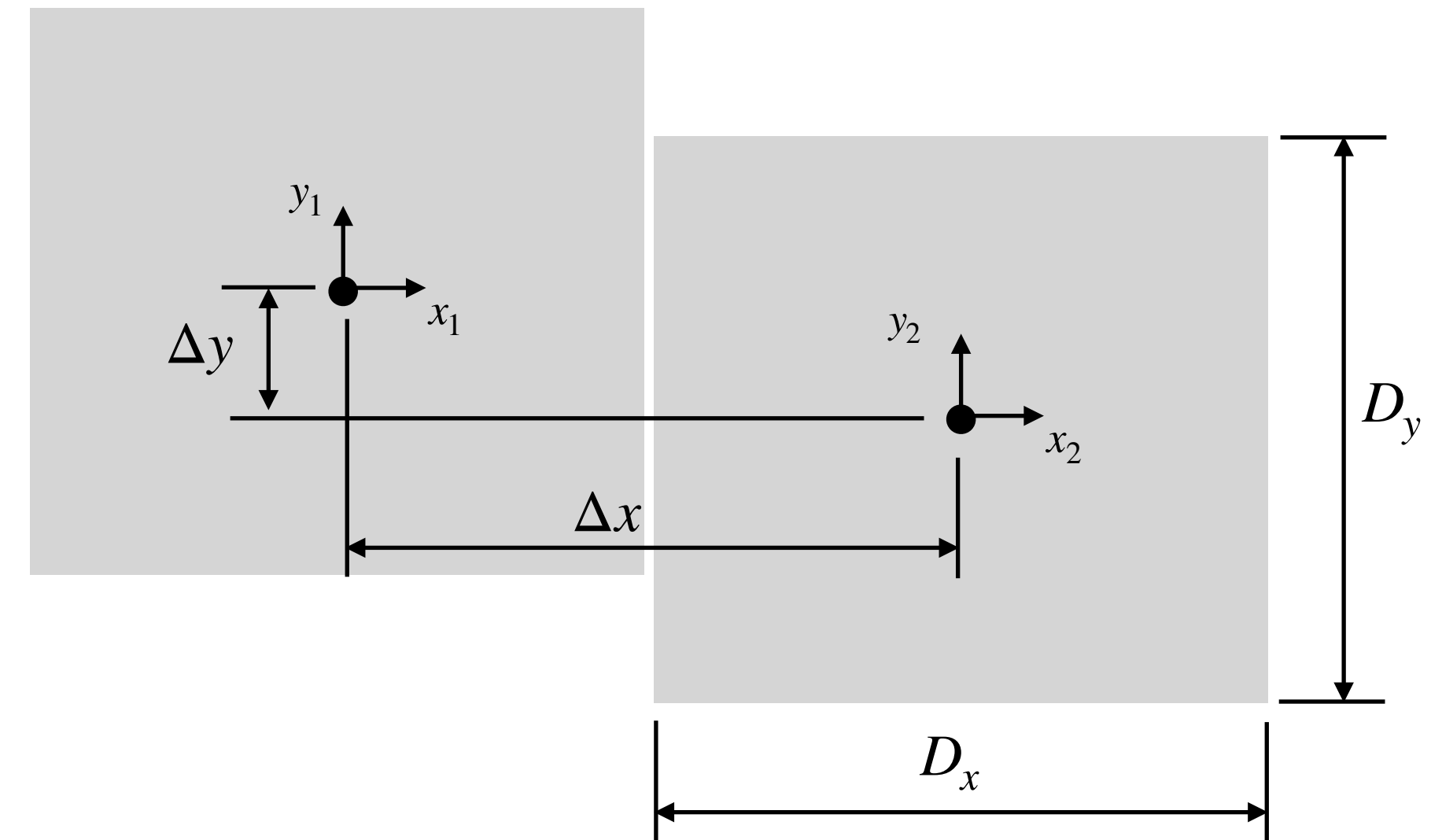
Effective inter-cell correlations

Different buildings,
different positions

$$\rho(\{T_1, \mathbf{x}_1\}, \{T_2, \mathbf{x}_2\}) < 1$$



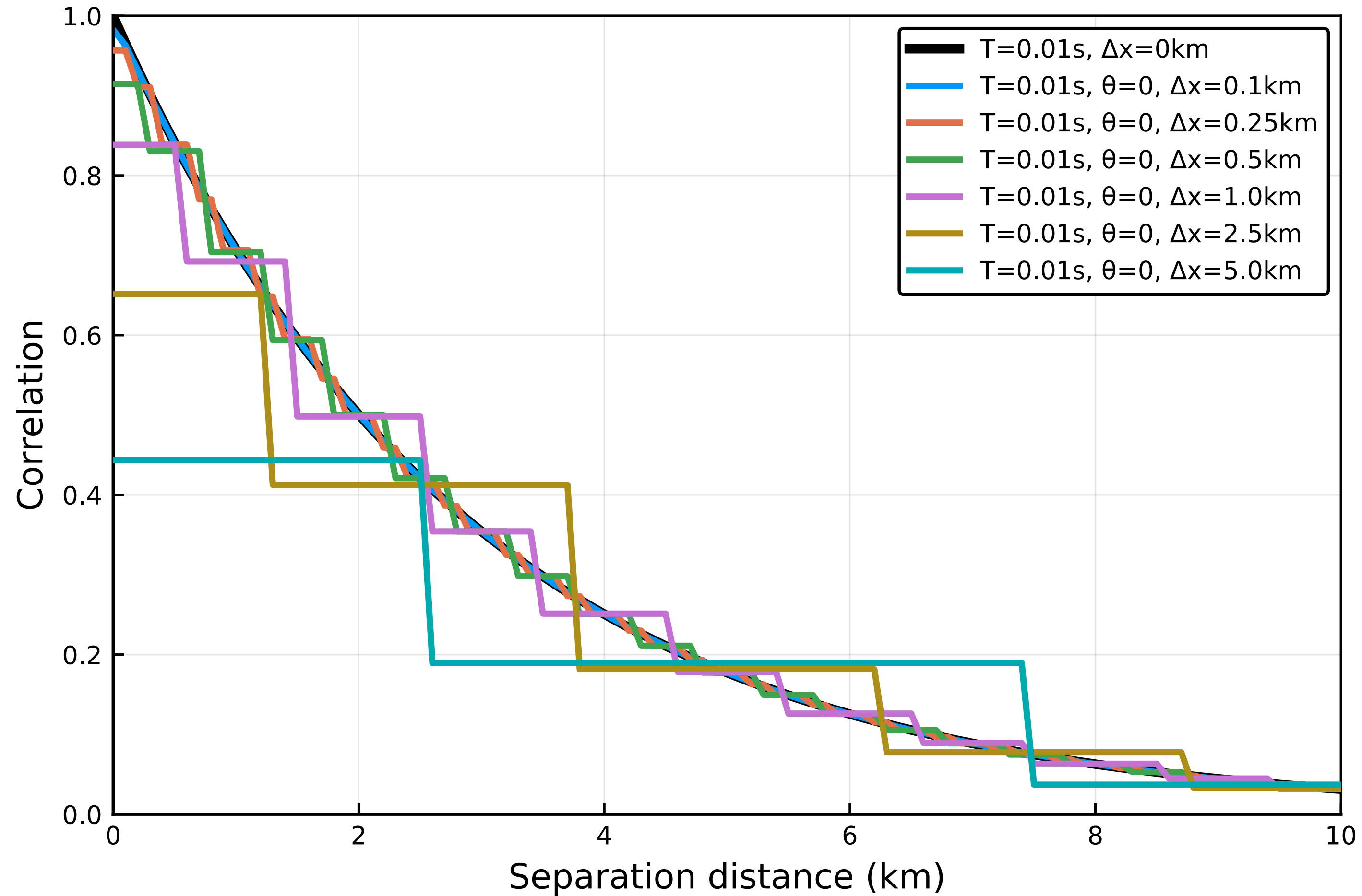
- Similarly to the effective inter-period correlations, we make the extension of the spatial discretisation over offset cells
- This reflects the unknown location of buildings of different classes throughout the respective cells



Have to consider possible
combinations of actual
locations for each building

$$\rho_{eff}(T_1, T_2) = \frac{1}{D_x^2 D_y^2} \iiint_{+=\Delta x, +=\Delta y} \rho(\{x_1, y_1\}, \{x_2, y_2\}, T_1, T_2) dx_1 dx_2 dy_1 dy_2$$

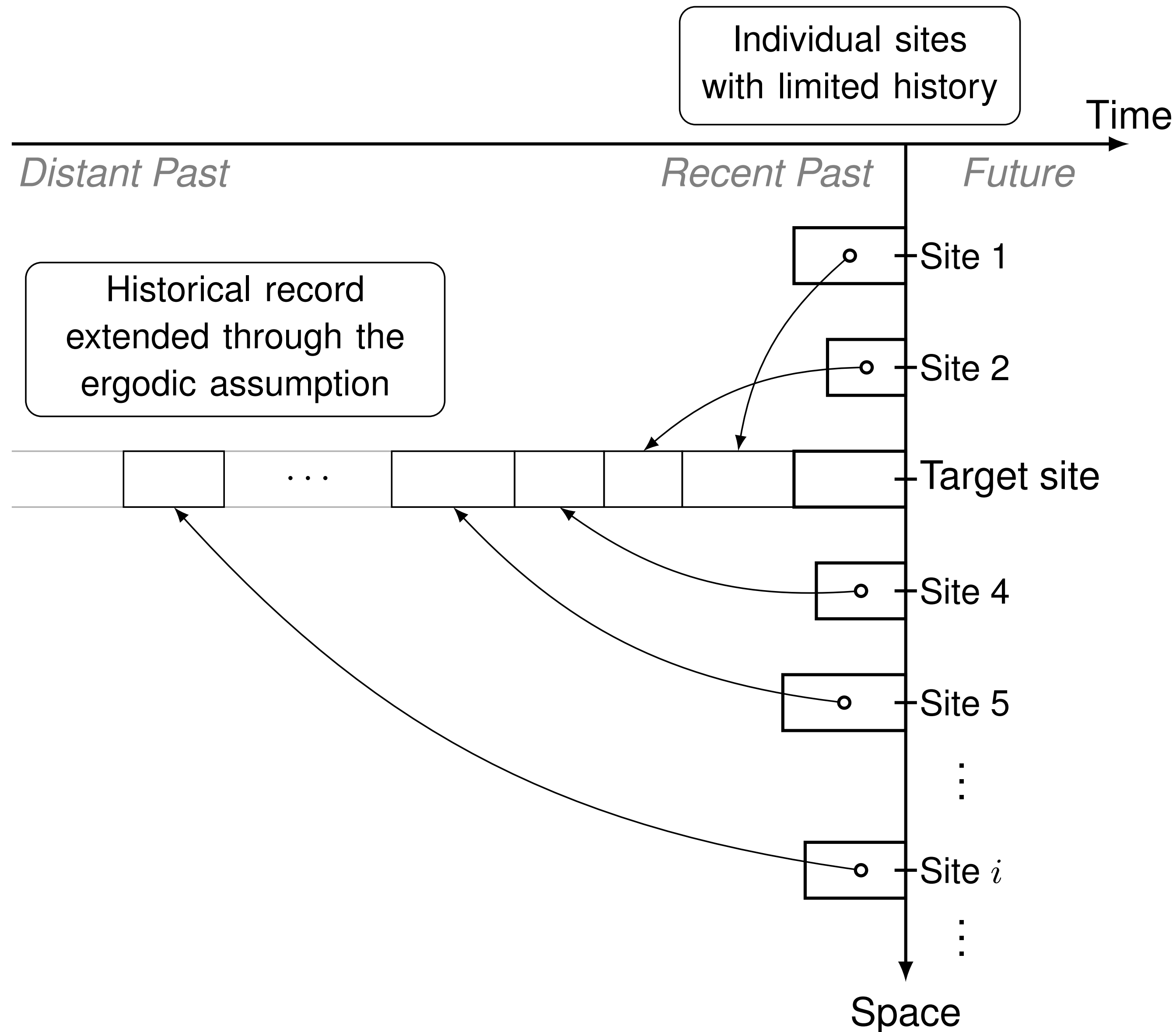
Effective inter-cell correlations



- Largest differences arise at short separation distances where the correlation is most important
- Typically a small reduction in correlation, but it varies with the relative sizes of the cells

The ergodic assumption in ground-motion modelling

Ergodic assumption



- We desire a ground-motion model that reflects the local source, path and site characteristics
- At the same time we want a model that robustly predicts ground-motions for all rupture scenarios of interest in the risk analysis
- We can't have both, so we favour the second point by importing data from other regions
- We assume that this pooled (ergodic) data have the same statistical characteristics as what would be obtained at our target site
- We effectively trade space for time
- In doing so we inflate the variance and over-estimate correlations

Components of ground motion variability

General representation of variability components in ground-motion models for portfolio risk applications

Simulated surface
motion (linear case)

Location
variate

Between-event
variate

Component-to-
component variate

$$im(\mathbf{x}_s) = \mu(\mathbf{x}_s; rup) + \delta_L(\mathbf{x}_e) + \delta_B + \delta_{W_{es}}(\mathbf{x}_s) + \delta_{S2S}(\mathbf{x}_s) + \delta_{C2C}(\mathbf{x}_s)$$

Median surface
prediction (linear
case)

Site & Event
corrected within-
event variate

Between-site
variate

Nonlinear site response complicates this a little as we first predict an motion at some reference velocity horizon (at depth), and then compute the amplification as a function of this motion

Motion at depth: $im_{ref}(\mathbf{x}) = \mu_{ref}(\mathbf{x}; rup) + \delta_L(\mathbf{x}) + \delta_B + \delta_{W_{es}}(\mathbf{x}) + \delta_{C2C}(\mathbf{x})$

Motion at surface: $im(\mathbf{x}) = im_{ref}(\mathbf{x}) + \mu_{\ln AF} \left[\mathbf{x}, im_{ref}(\mathbf{x}) \right] + \delta_{S2S} \left[\mathbf{x}; im_{ref}(\mathbf{x}) \right]$

Components of GM variability

Traditional ergodic approach

$$im(\mathbf{x}) = \mu(\mathbf{x}; rup) + \delta_B + \delta_W(\mathbf{x})$$

$$\Sigma(\mathbf{x}; rup) \equiv \tau^2 + \phi^2(\mathbf{x})$$

Non-ergodic approach

$$im(\mathbf{x}) = \mu(\mathbf{x}; rup) + \underbrace{\delta_L(\mathbf{x}_e) + \delta_B}_{\delta_B \text{ ergodic equivalents}} + \underbrace{\delta_{S2S}(\mathbf{x}_s) + \delta_{W_{es}}(\mathbf{x}_s)}_{\delta_W(\mathbf{x})}$$

$$\Sigma(\mathbf{x}; rup) \equiv \underbrace{\tau_L^2(\mathbf{x}_e) + \tau_B^2}_{\tau^2 \text{ ergodic equivalents}} + \underbrace{\phi_{SS}^2(\mathbf{x}_s) + \phi_{S2S}^2(\mathbf{x}_s)}_{\phi^2(\mathbf{x})}$$

The systematic source effect $\delta_L(\mathbf{x}_e)$ and systematic site effect $\delta_{S2S}(\mathbf{x}_s)$ are actually **epistemic** terms that should be reflected within a logic tree rather than in the aleatory variability

Components of GM variability

Non-ergodic approach

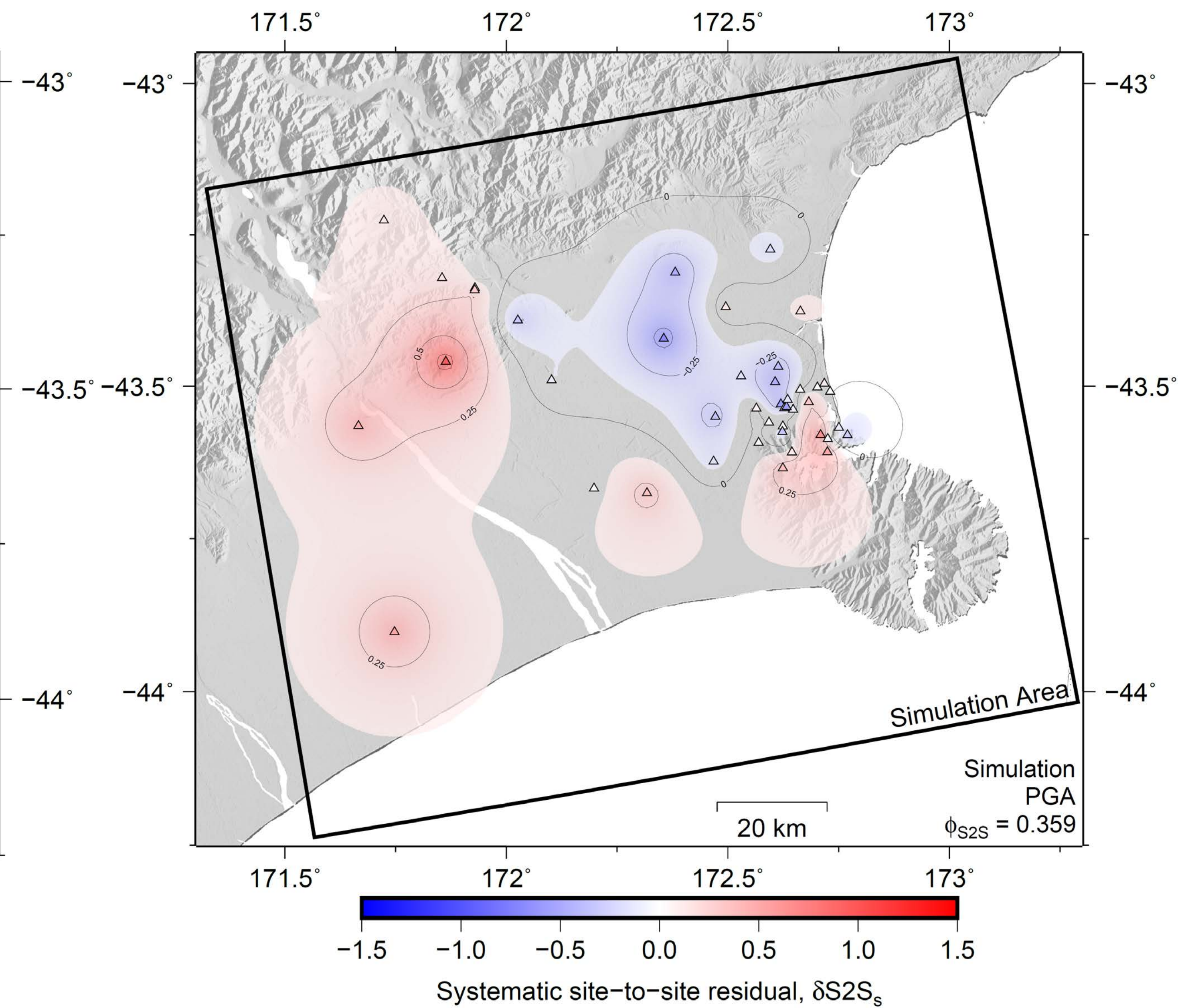
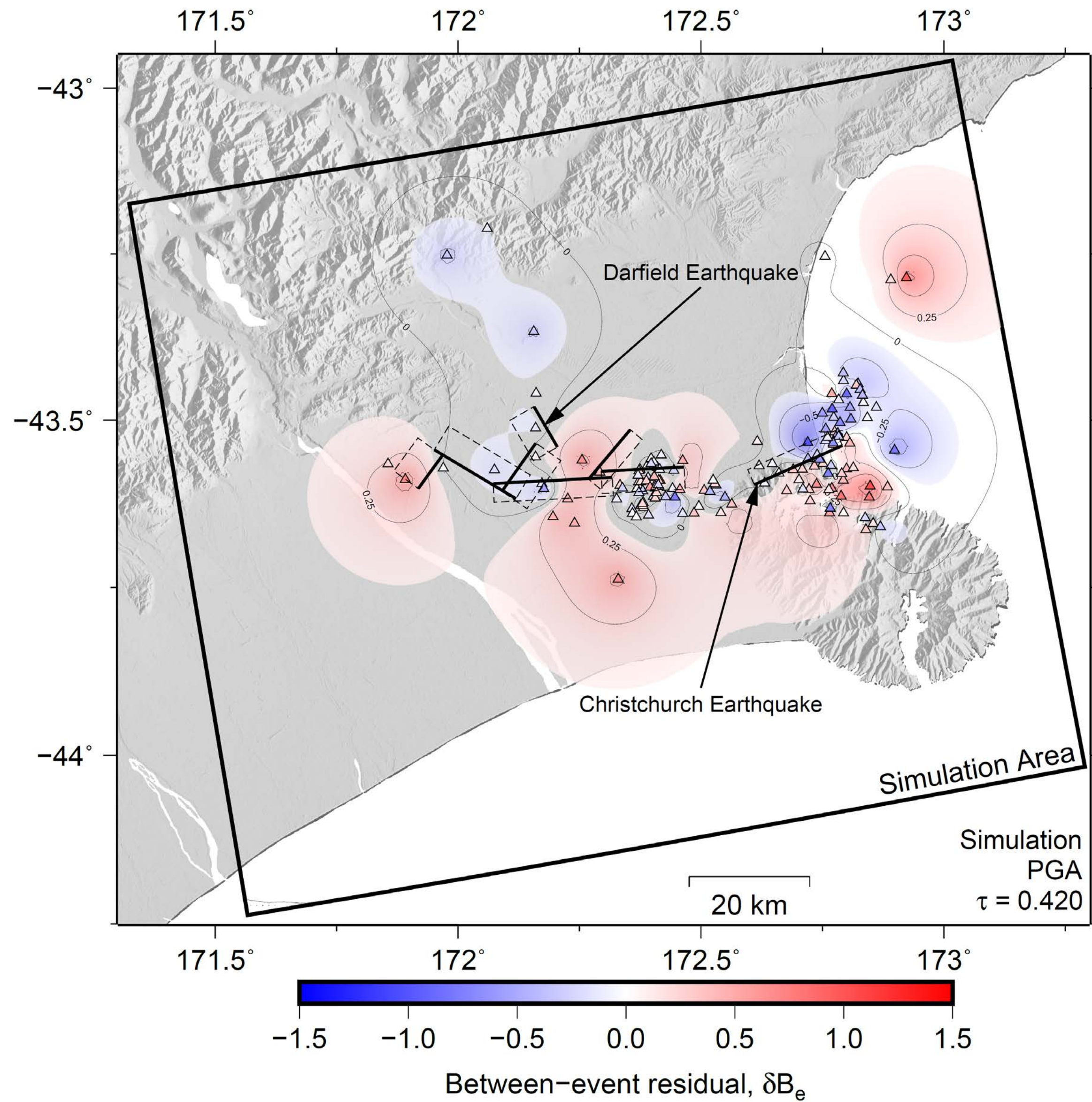
$$im(\mathbf{x}) = \mu(\mathbf{x}; rup) + \underbrace{\hat{\delta}_L(\mathbf{x}_e) + \delta_B}_{\delta_B \text{ ergodic equivalents}} + \underbrace{\hat{\delta}_{S2S}(\mathbf{x}_s) + \delta_{W_{es}}(\mathbf{x}_s)}_{\delta_W(\mathbf{x})}$$

$$\Sigma(\mathbf{x}; rup) \equiv \underbrace{\tau_L^2 \cancel{\mathbf{x}_e}}_{\tau^2 \text{ ergodic equivalents}} + \tau_B^2 + \underbrace{\phi_{SS}^2(\mathbf{x}_s) + \phi_{S2S}^2 \cancel{\mathbf{x}_s}}_{\phi^2(\mathbf{x})}$$

With the systematic source effect $\delta_L(\mathbf{x}_e)$ and systematic site effect $\delta_{S2S}(\mathbf{x}_s)$ estimated, the covariance terms reduce as $\tau_B^2 < \tau^2$ and $\phi_{SS}^2(\mathbf{x}) < \phi^2(\mathbf{x})$

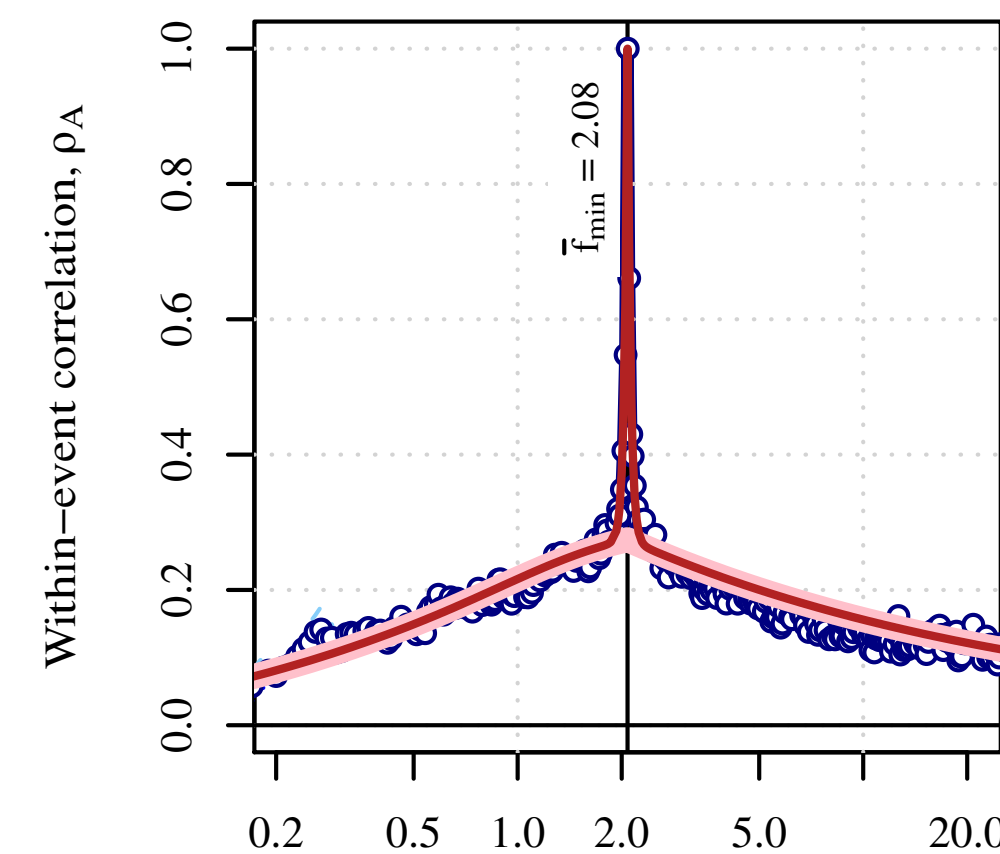
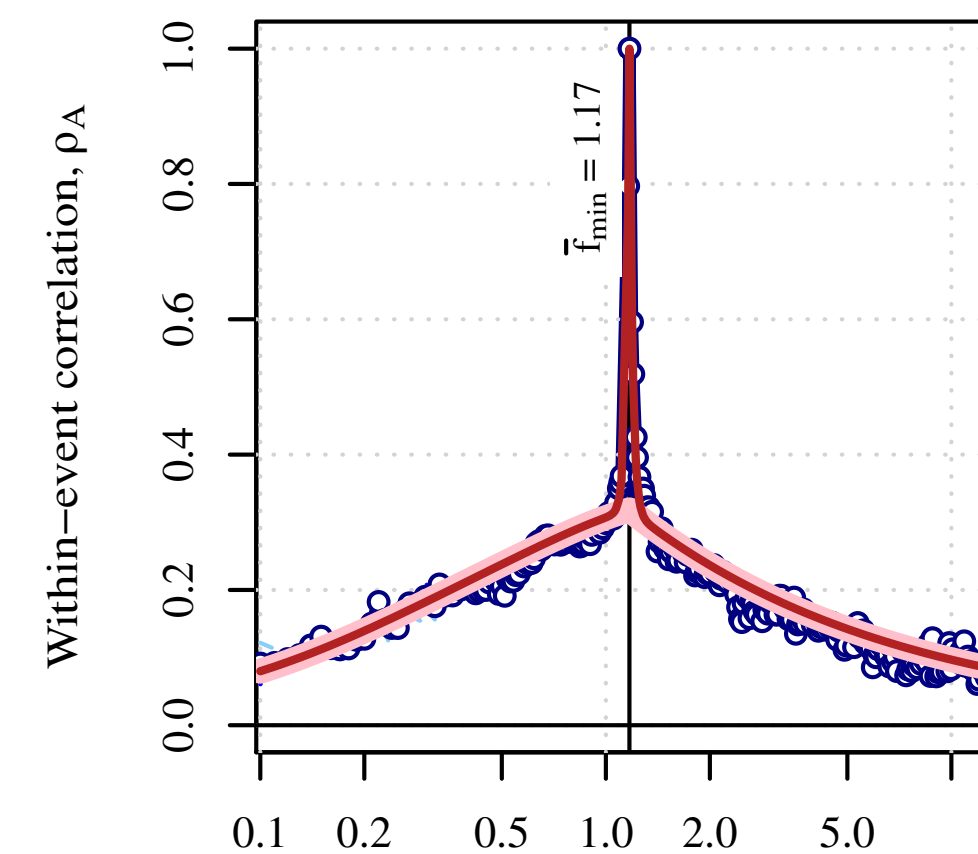
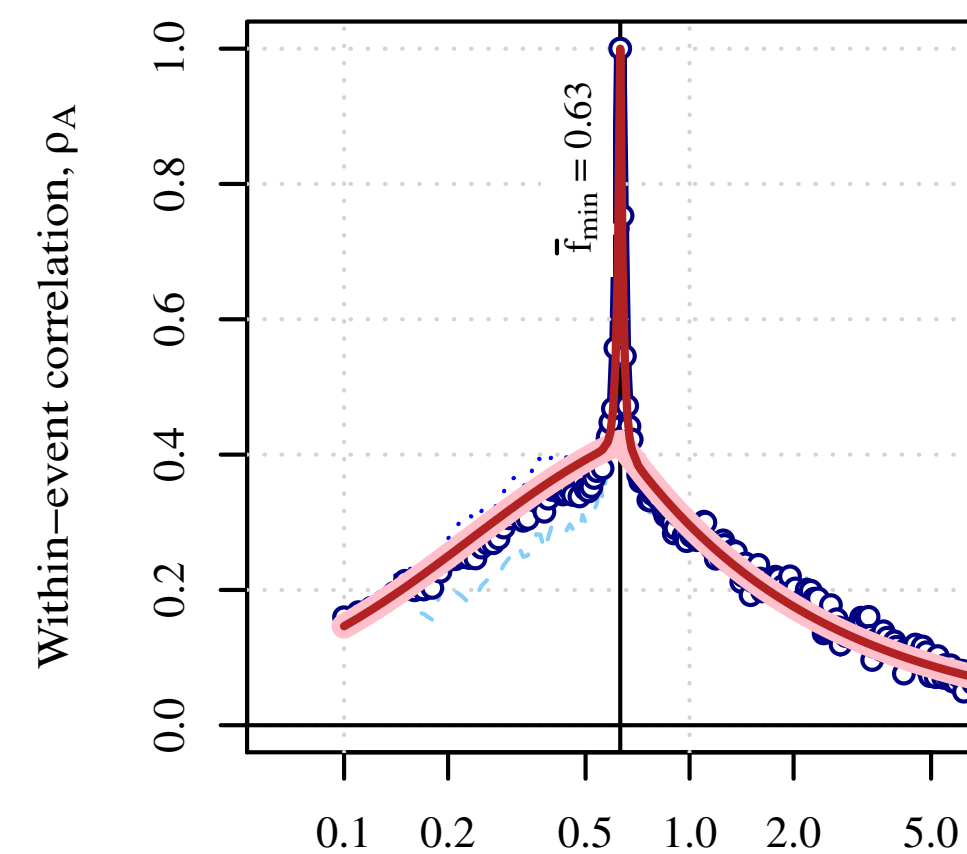
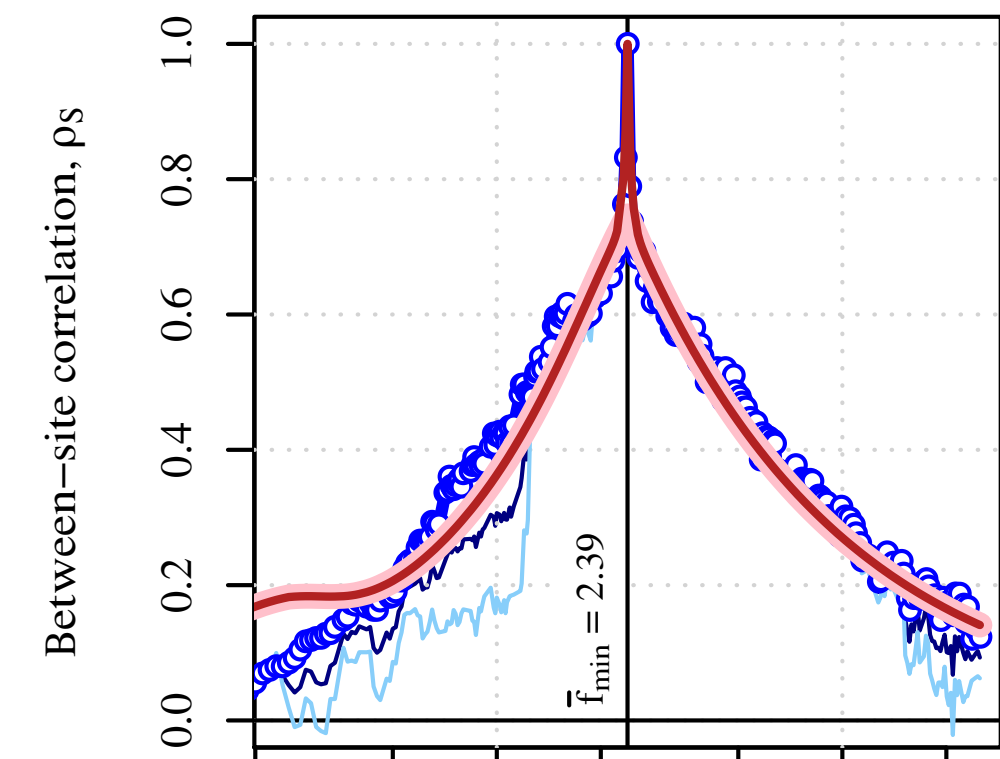
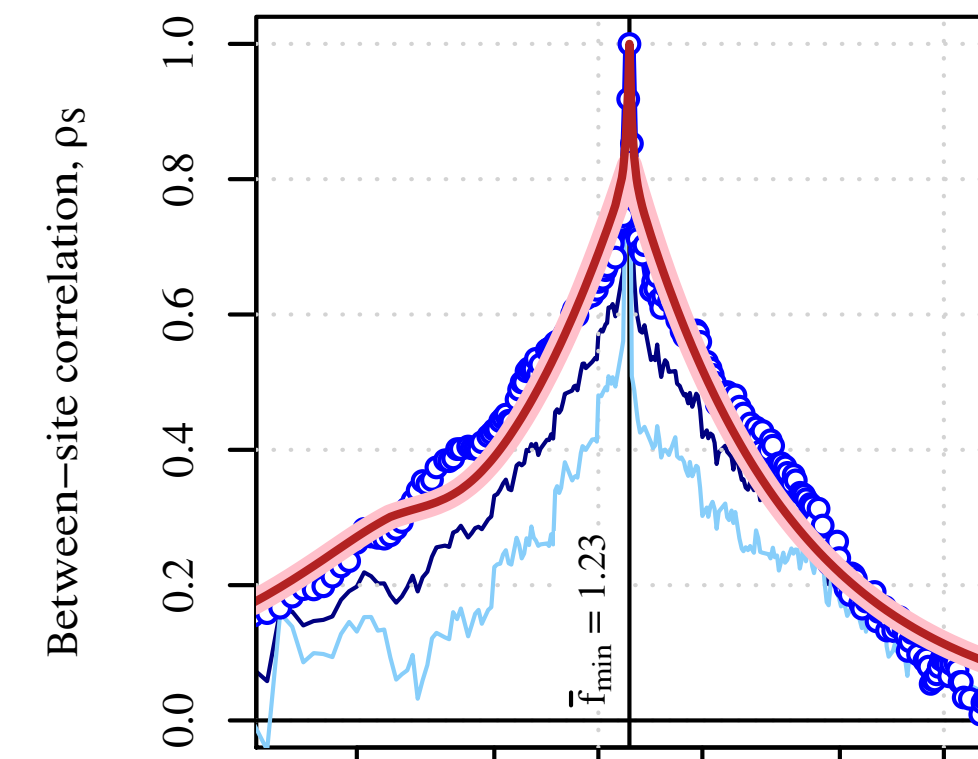
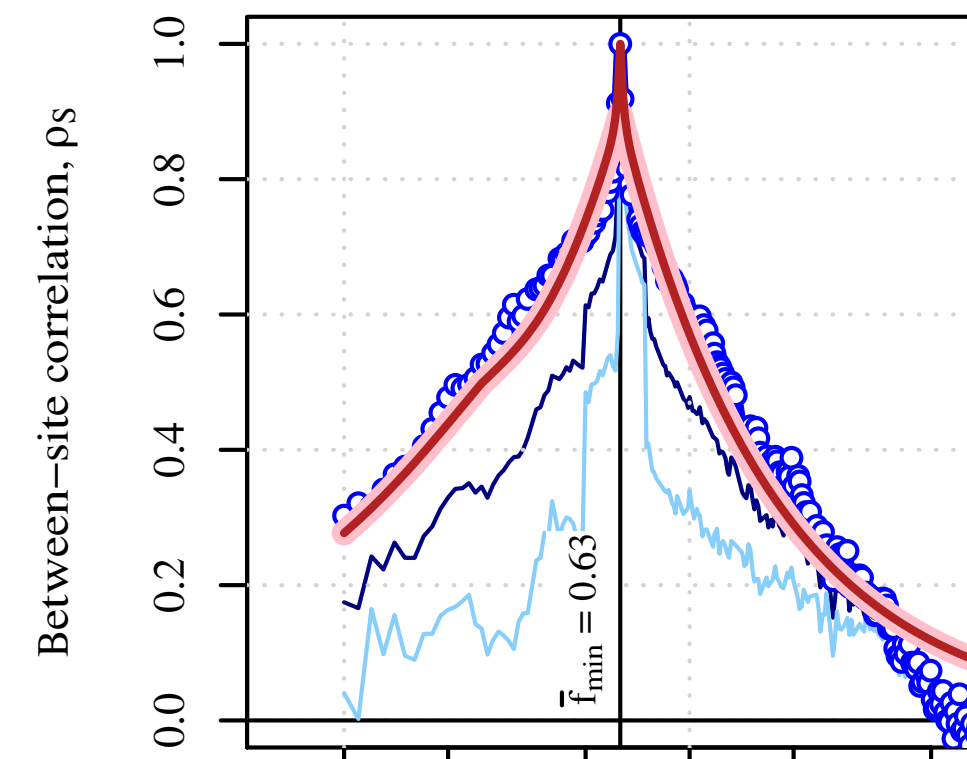
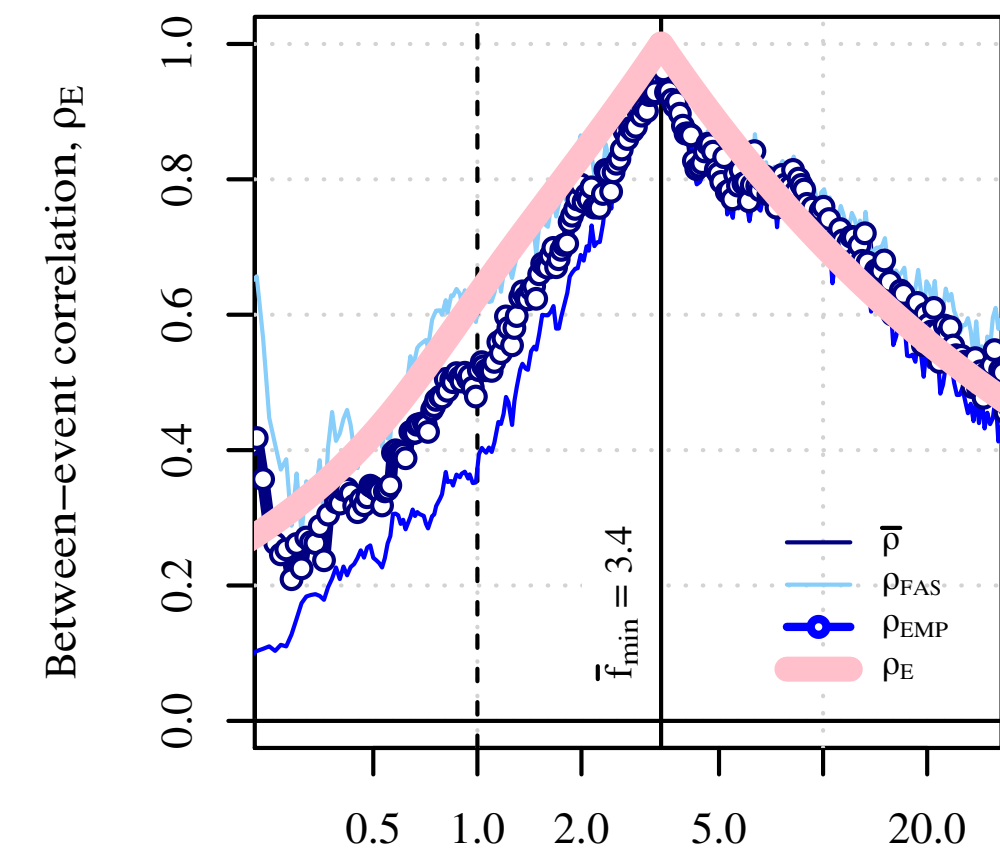
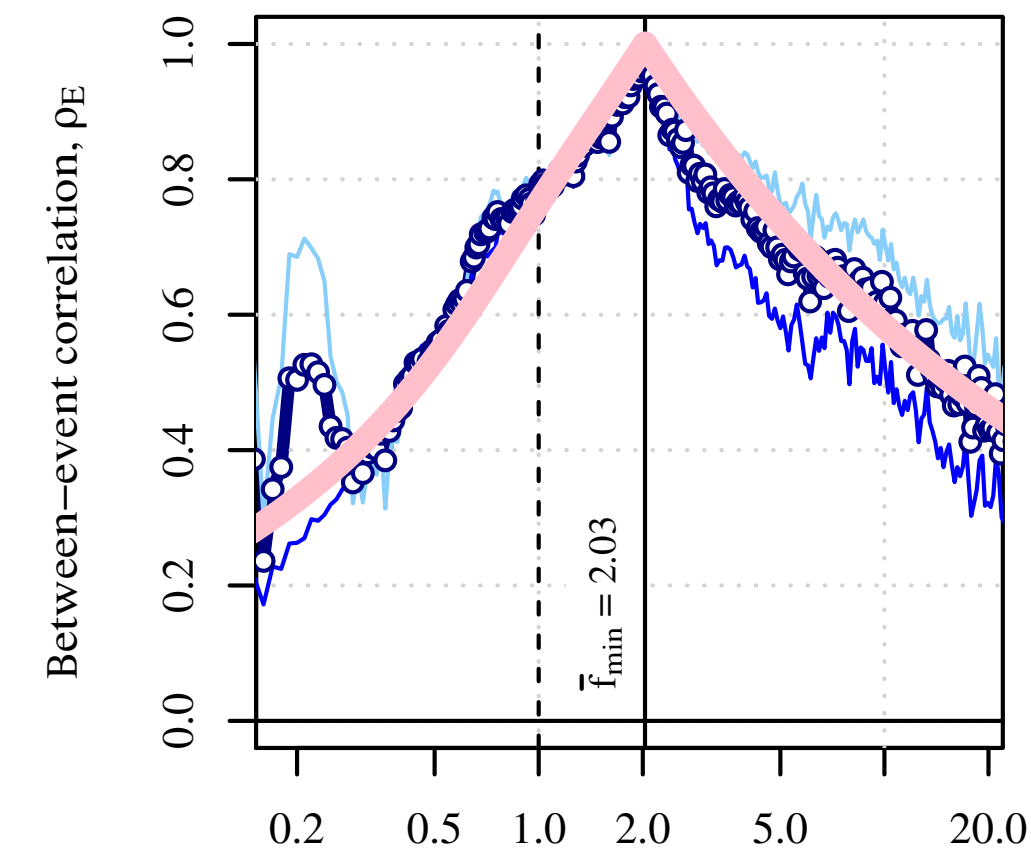
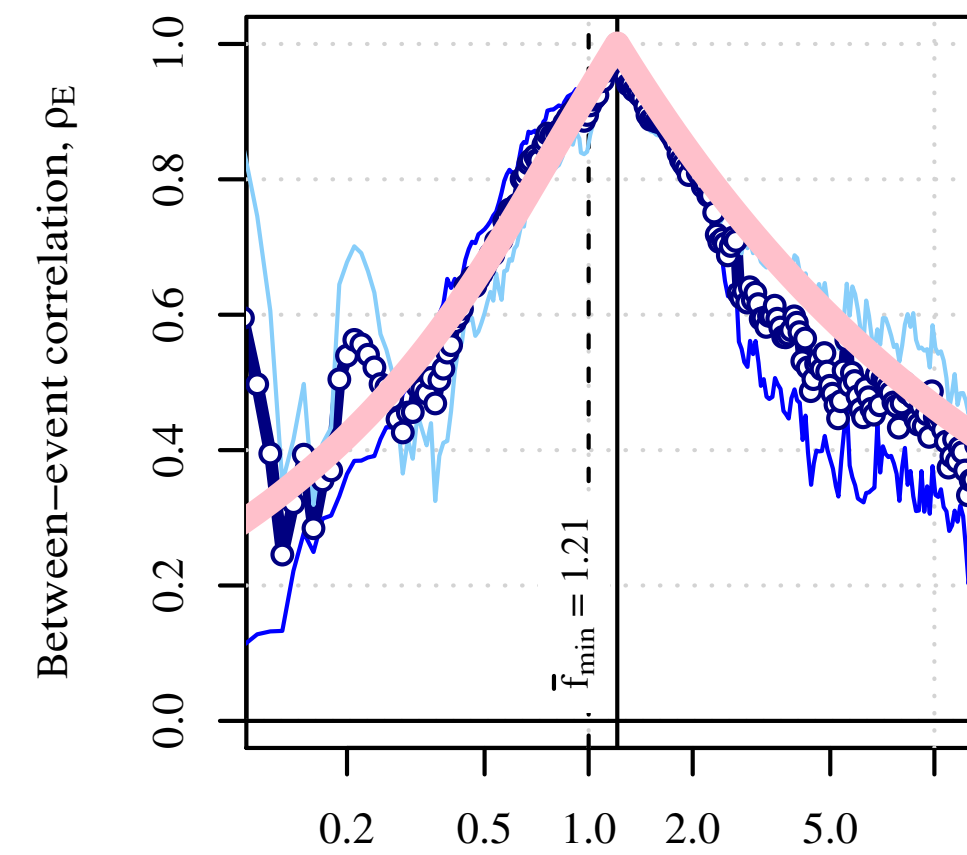
As noted yesterday, Istanbul is exposed to a very particular rupture scenario, and relatively good information exists regarding the near-surface geotechnical profiles

Systematic source & site effects



Impact upon inter-period correlation

- Here inter-frequency correlations among Fourier spectral ordinates are shown
- Correlations among the between-site components is relatively strong and so the removal of these effects reduces the overall correlation
- This maps through to inter-period correlations among response spectral ordinates



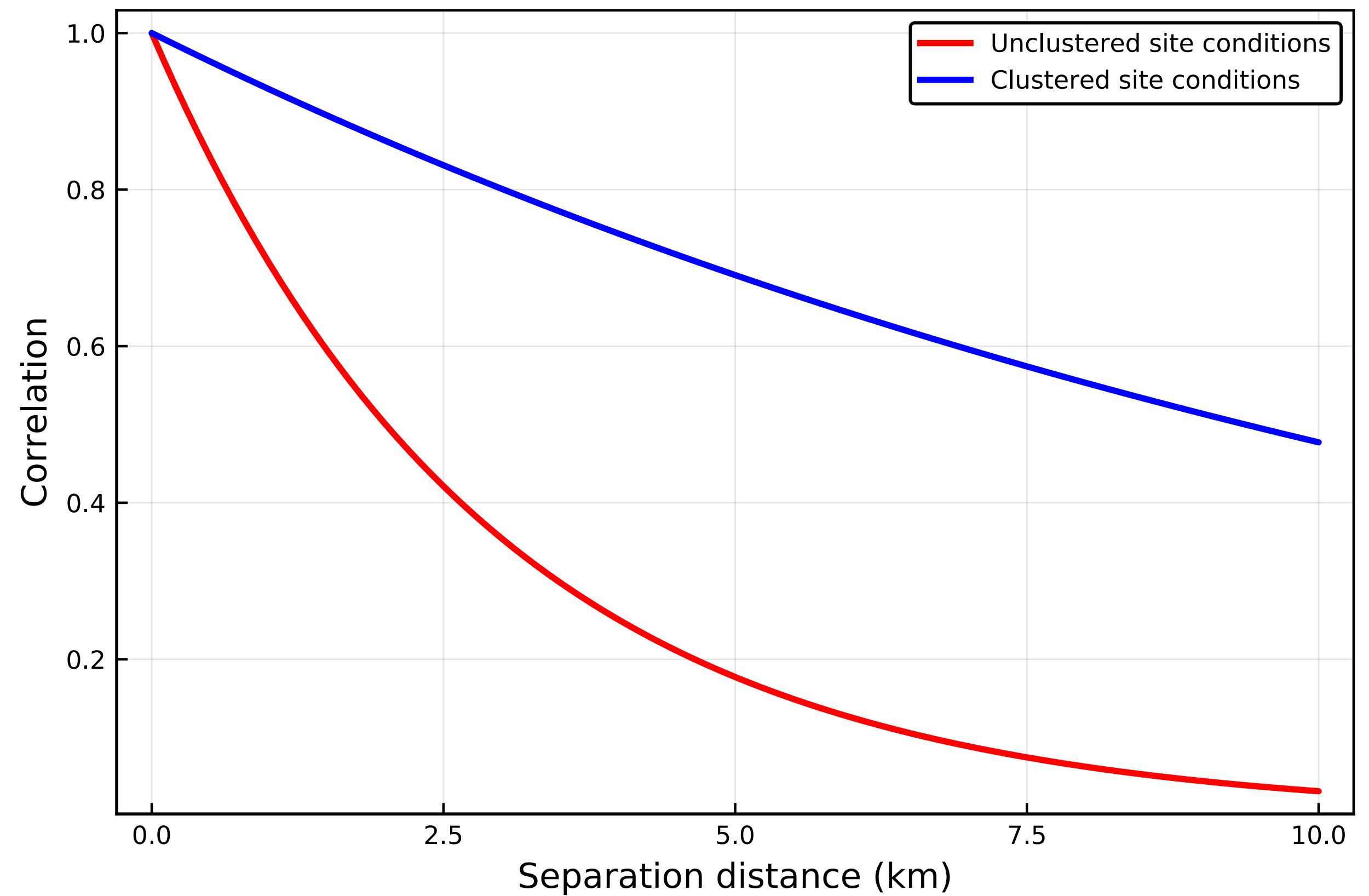
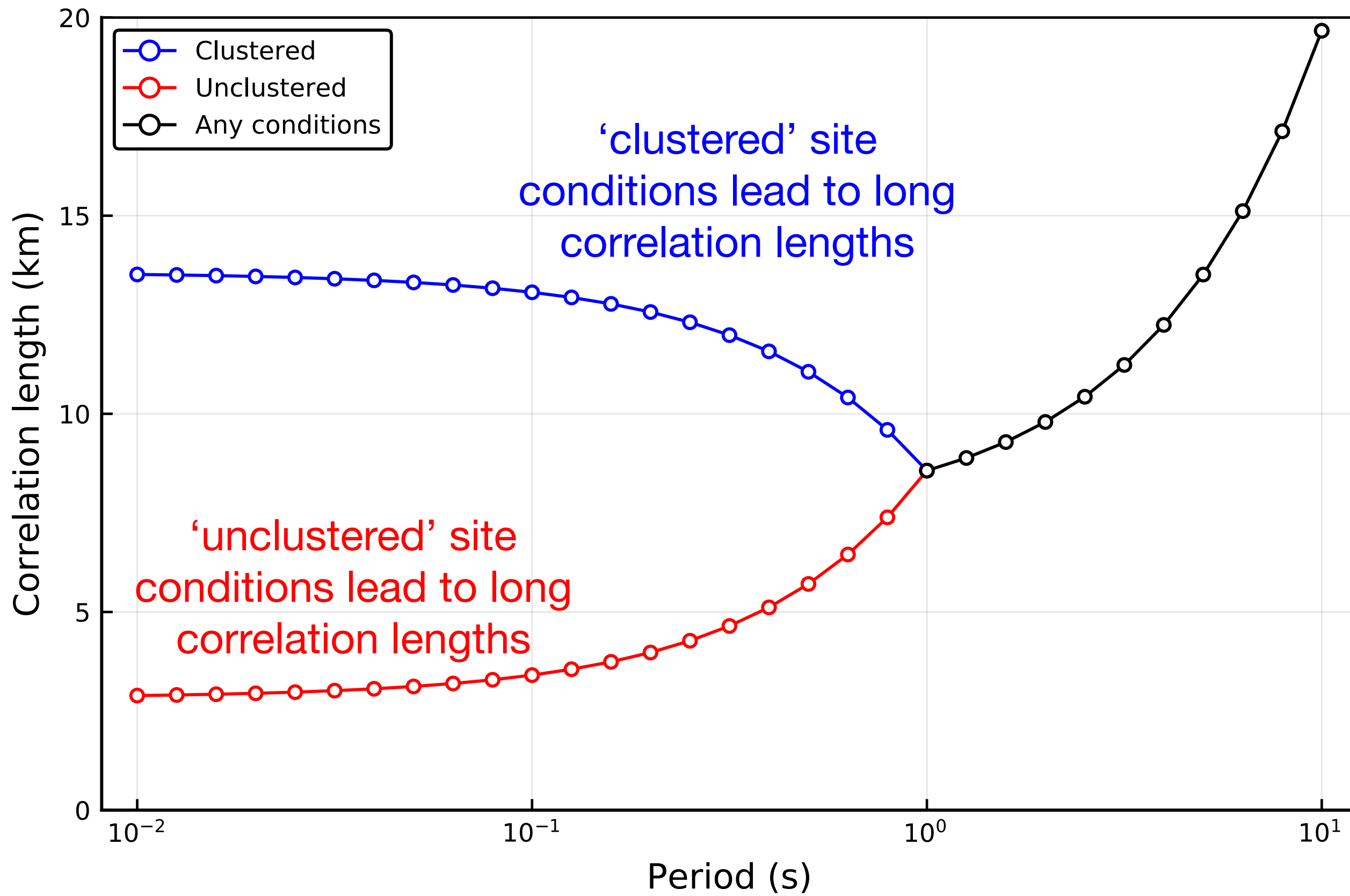
Impact upon spatial correlation

- Jayaram & Baker (2009) presented a now widely-adopted spatial correlation model
- They use an exponential model that has a different correlation length, r , for periods below 1.0 seconds, depending upon whether site conditions are ‘clustered’ or ‘unclustered’
- This approach was motivated by strongly different correlation lengths found in different regions (and soil conditions were held responsible)
- Their model is **ergodic** and can be expressed as:

$$\ln Sa(\mathbf{x}) = \mu_{\ln Sa}(\mathbf{x}; rup) + \delta_B + \delta_W(\mathbf{x}) \quad \delta_B \sim N(0, \tau^2) \quad \delta_W(\mathbf{x}) \sim MVN(\mathbf{0}, \phi^2(\mathbf{x})\mathbf{R}(\mathbf{x}))$$

$$\rho\left(\delta_W(\mathbf{x}_i), \delta_W(\mathbf{x}_j)\right) = \exp\left(-\frac{||\mathbf{x}_i - \mathbf{x}_j||}{r}\right) = \exp\left(-\frac{\Delta}{r}\right) = \rho(\Delta)$$

Ergodic spatial correlation



Non-ergodic spatial correlation

- If we partition the variance so that we have systematic event and site effects then we rewrite the model as:

$$\ln Sa(\mathbf{x}) = \mu_{\ln Sa}(\mathbf{x}; rup) + \delta_B + \delta_{S2S}(\mathbf{x}) + \delta_{W_{es}}(\mathbf{x})$$

$$\delta_B \sim N(0, \tau^2)$$

$$\delta_{S2S}(\mathbf{x}) \sim MVN[\mathbf{0}, \phi_{S2S}^2(\mathbf{x})\mathbf{R}_S(\mathbf{x})]$$

$$\delta_{W_{es}}(\mathbf{x}) \sim MVN[\mathbf{0}, \phi_{SS}^2(\mathbf{x})\mathbf{R}_W(\mathbf{x})]$$

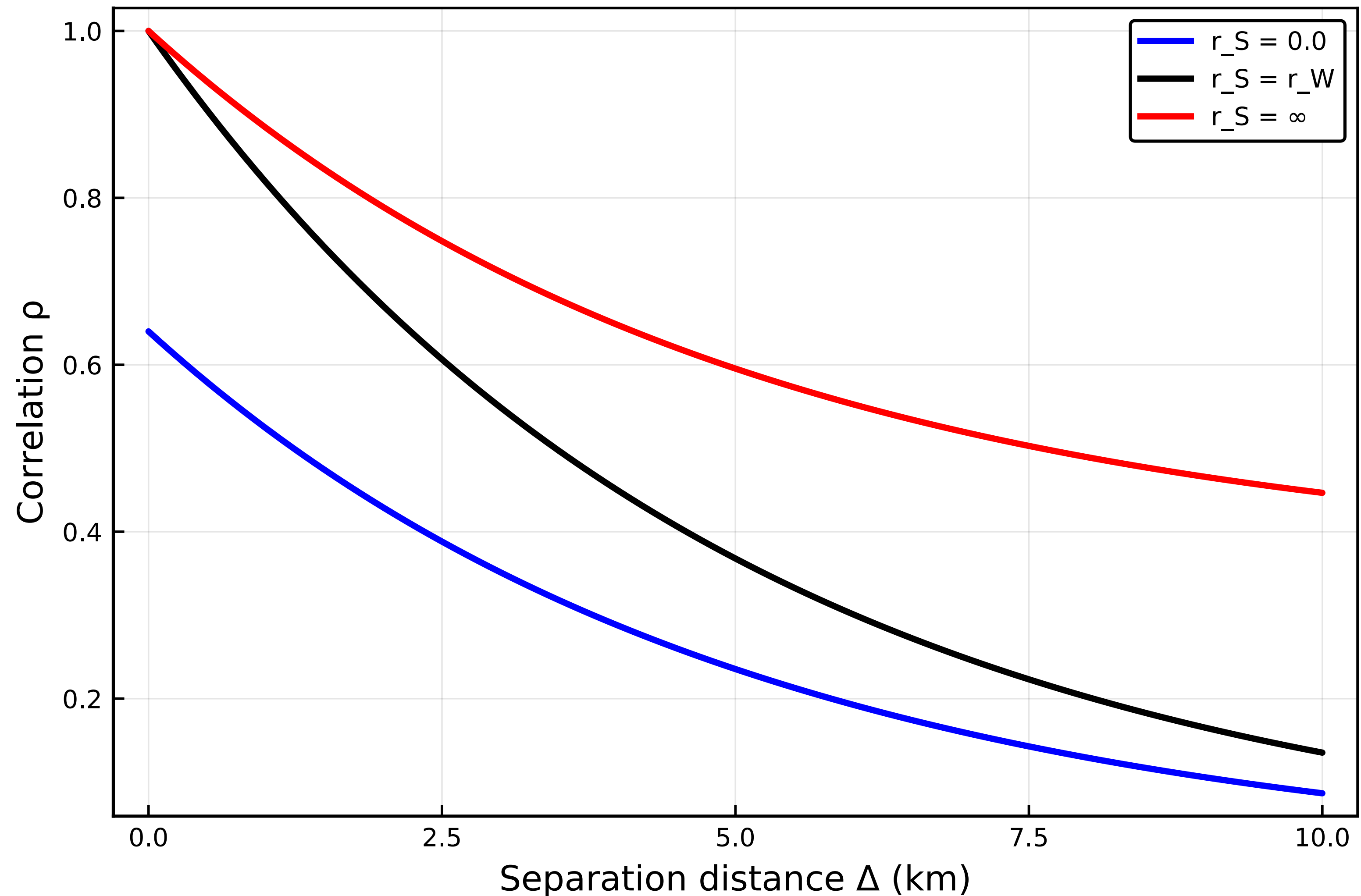
- The overall within-event spatial correlation in this case is represented by:

$$\rho[\delta_W(\mathbf{x}_i), \delta_W(\mathbf{x}_j)] = \frac{\rho_W(\mathbf{x}_i, \mathbf{x}_j)\phi_{SS}(\mathbf{x}_i)\phi_{SS}(\mathbf{x}_j) + \rho_S(\mathbf{x}_i, \mathbf{x}_j)\phi_{S2S}(\mathbf{x}_i)\phi_{S2S}(\mathbf{x}_j)}{\phi(\mathbf{x}_i)\phi(\mathbf{x}_j)} \quad \phi^2 = \phi_{SS}^2 + \phi_{S2S}^2$$

- Express the spatial correlations among δ_{S2S} terms and among $\delta_{W_{es}}$ terms as exponential models

$$\rho_S(\mathbf{x}_i, \mathbf{x}_j) = \exp\left(-\frac{\Delta}{r_S}\right) \quad \rho_W(\mathbf{x}_i, \mathbf{x}_j) = \exp\left(-\frac{\Delta}{r_W}\right)$$

Non-ergodic spatial correlation



- We can therefore consider the ergodic correlation model as being contaminated by correlations among systematic site effects (epistemic terms) that have some correlation length r_S

$$\rho(\Delta) = \frac{1}{\phi^2} \left[\phi_{SS}^2 e^{-\frac{\Delta}{r_W}} + \phi_{S2S}^2 e^{-\frac{\Delta}{r_S}} \right]$$

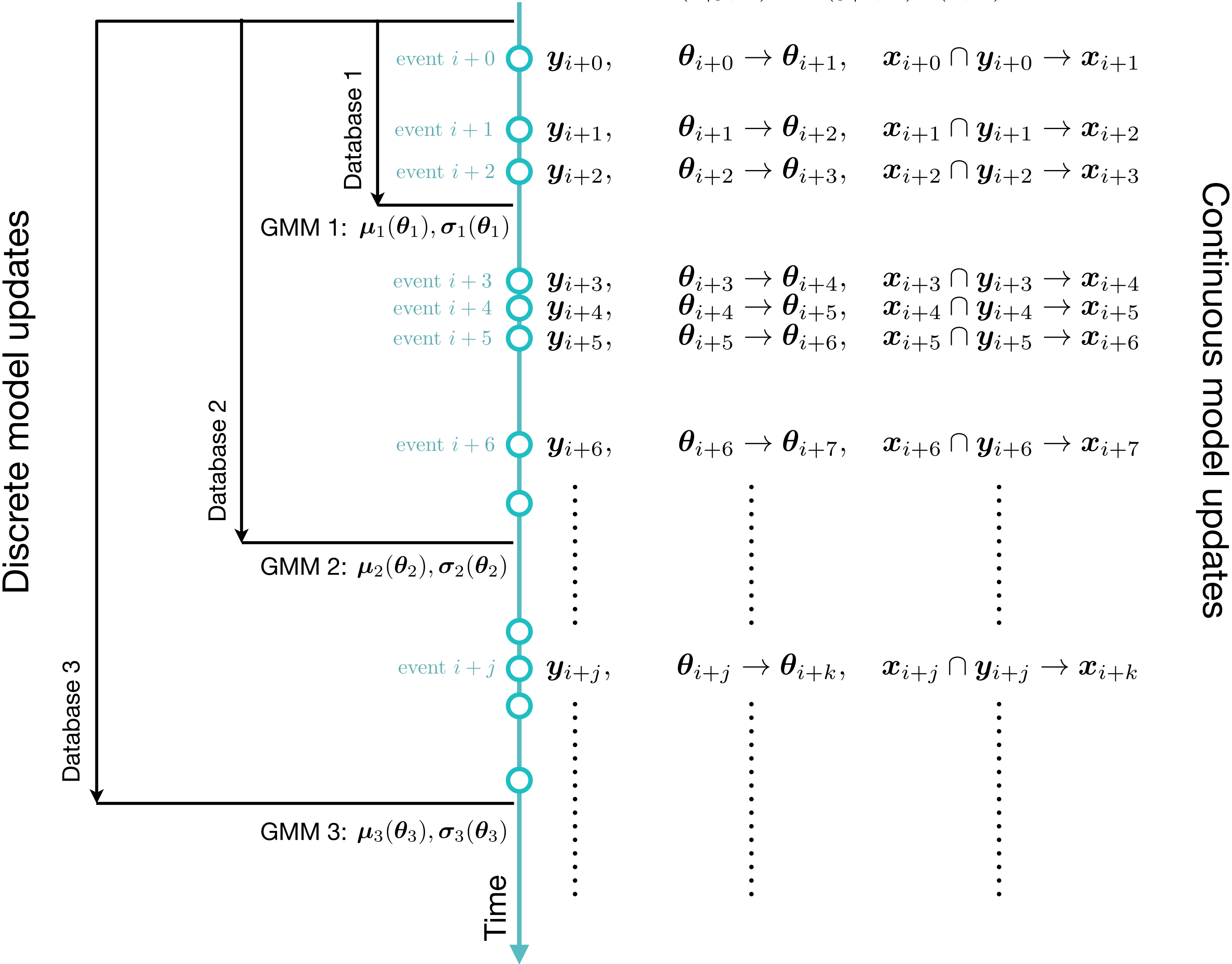
- Consider limiting cases of $r_S \rightarrow \infty$ (perfectly correlated site conditions) and $r_S \rightarrow 0$ (completely random site conditions)
- Jayaram & Baker's clustered vs non-clustered results just reflect implicit r_S values for the regions they investigated

Continuous updating of GMM

Traditional approach

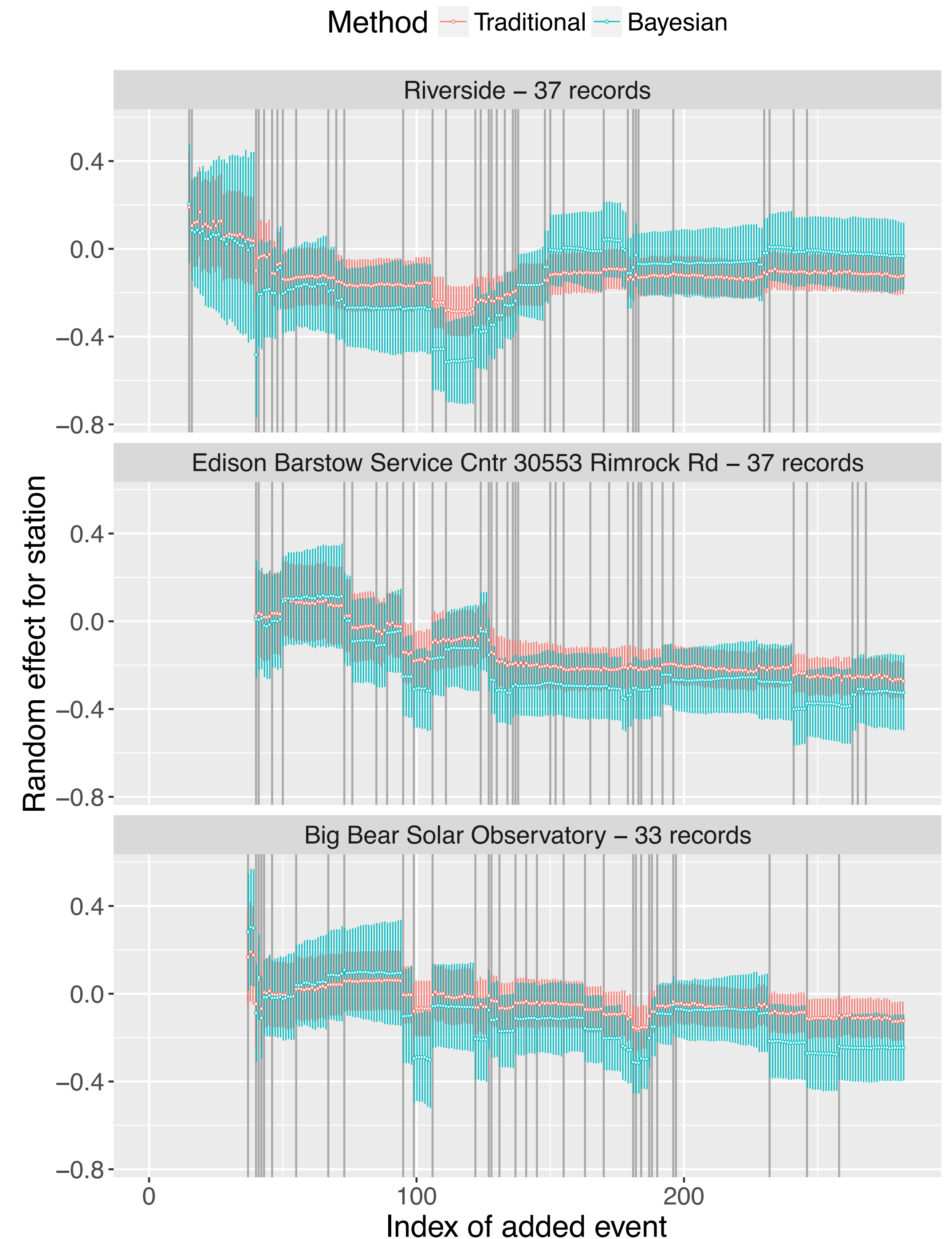
Bayesian approach

$$\mathcal{P}(\boldsymbol{\theta}|\boldsymbol{y}; \boldsymbol{x}) \propto \mathcal{L}(\boldsymbol{y}|\boldsymbol{\theta}; \boldsymbol{x})\mathcal{P}(\boldsymbol{\theta}; \boldsymbol{x})$$



Continuous update of systematic site effects

- In a given region, as every new event is recorded we can revise estimates of the systematic site effect
- For a risk model this would have implications upon the amount of epistemic uncertainty that exists at each site
- This change in uncertainty should also cause updates in the spatial correlations among systematic site effects



Non-ergodic risk analyses for seismic sequences

Earthquake sequences

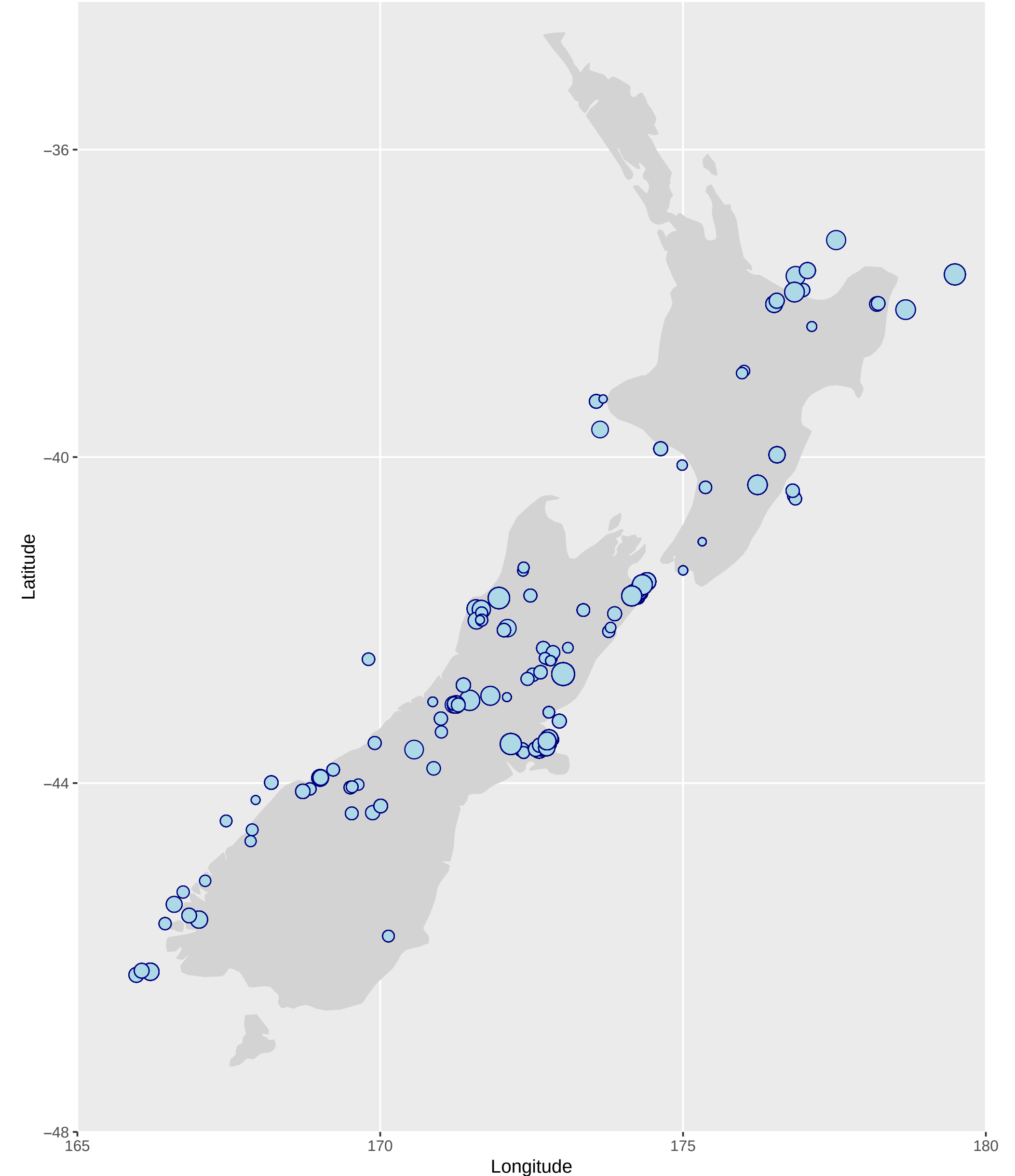
- Consider risk analysis for earthquake sequences - look at a couple of NZ examples
- Usually ongoing risk analyses continue to make use of generic ergodic ground-motion models

$$im(\mathbf{x}) = \mu(\mathbf{x}; rup) + \underbrace{\delta_L(\mathbf{x}_e) + \delta_C + \delta_{B,C} + \delta_{S2S}(\mathbf{x}_s)}_{\text{cluster-specific effect, and within-cluster source effect}} + \delta_{W_{es}}(\mathbf{x}_s)$$

cluster-specific effect, and
within-cluster source effect

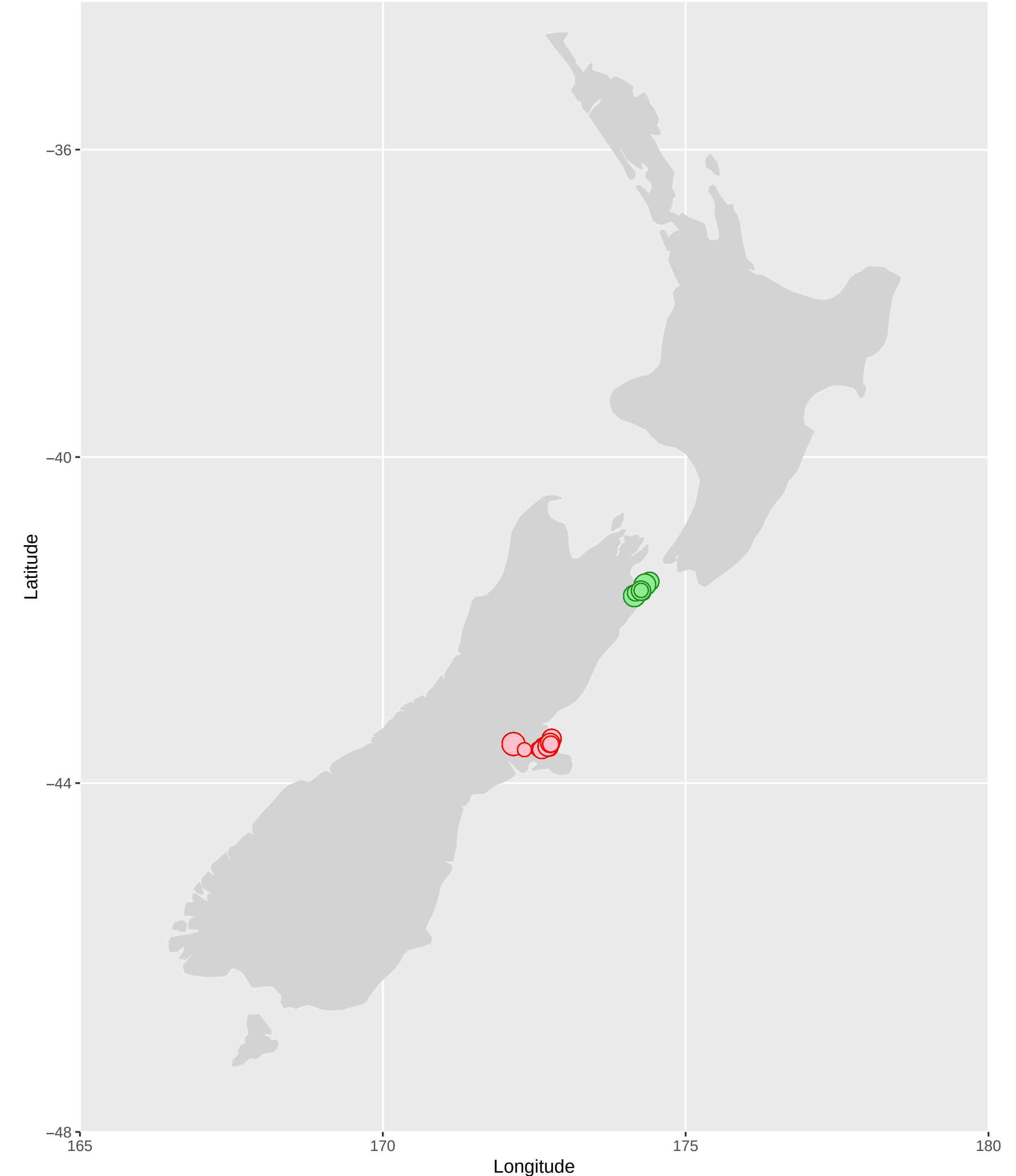
$$\Sigma(\mathbf{x}; rup) \equiv \tau_L^2(\mathbf{x}_e) + \underbrace{\tau_C^2 + \tau_{B,C}^2}_{\text{cluster-specific effect, and within-cluster source variability}} + \phi_{SS}^2(\mathbf{x}_s) + \phi_{S2S}^2(\mathbf{x}_s)$$

cluster-specific effect, and
within-cluster source variability



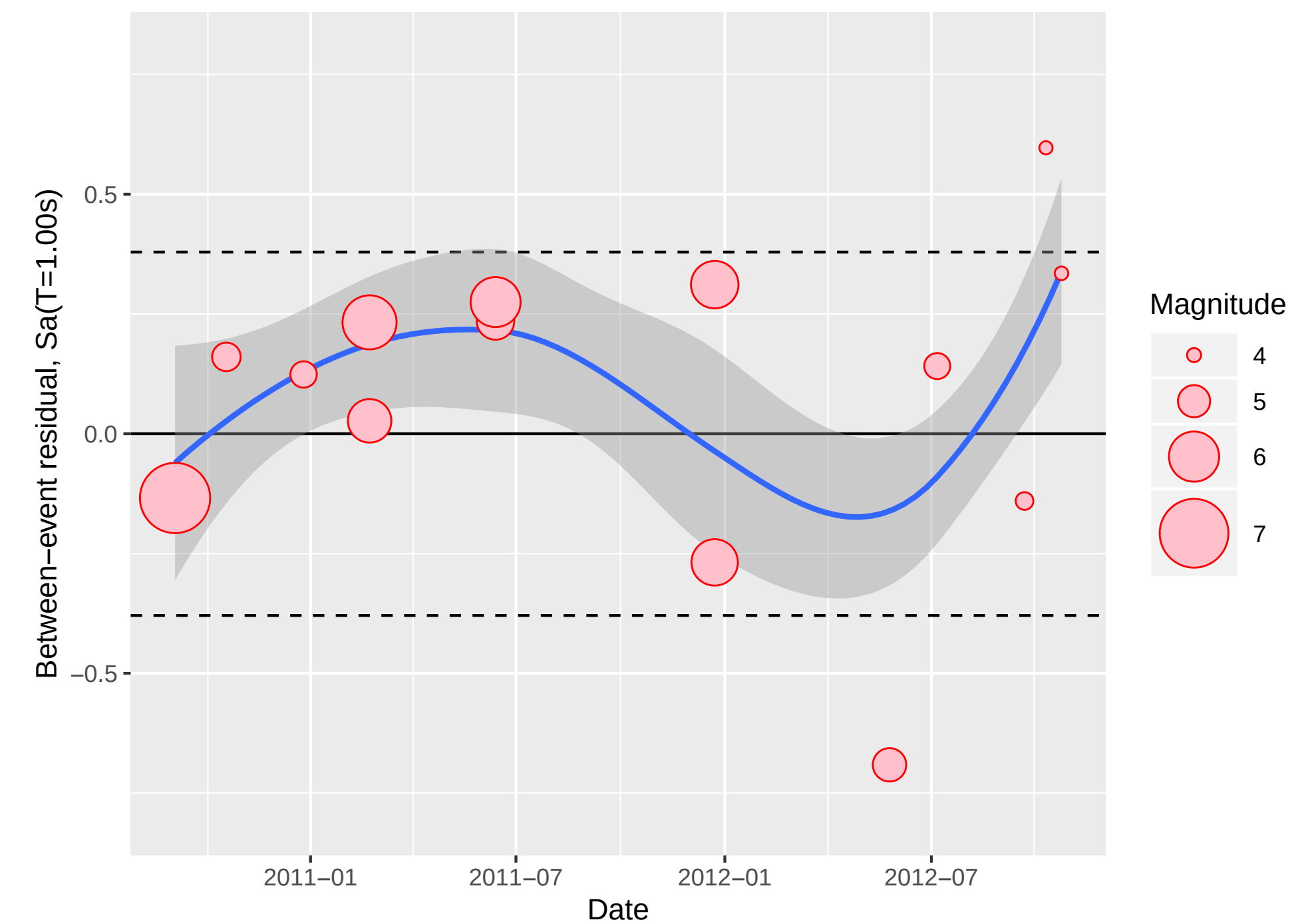
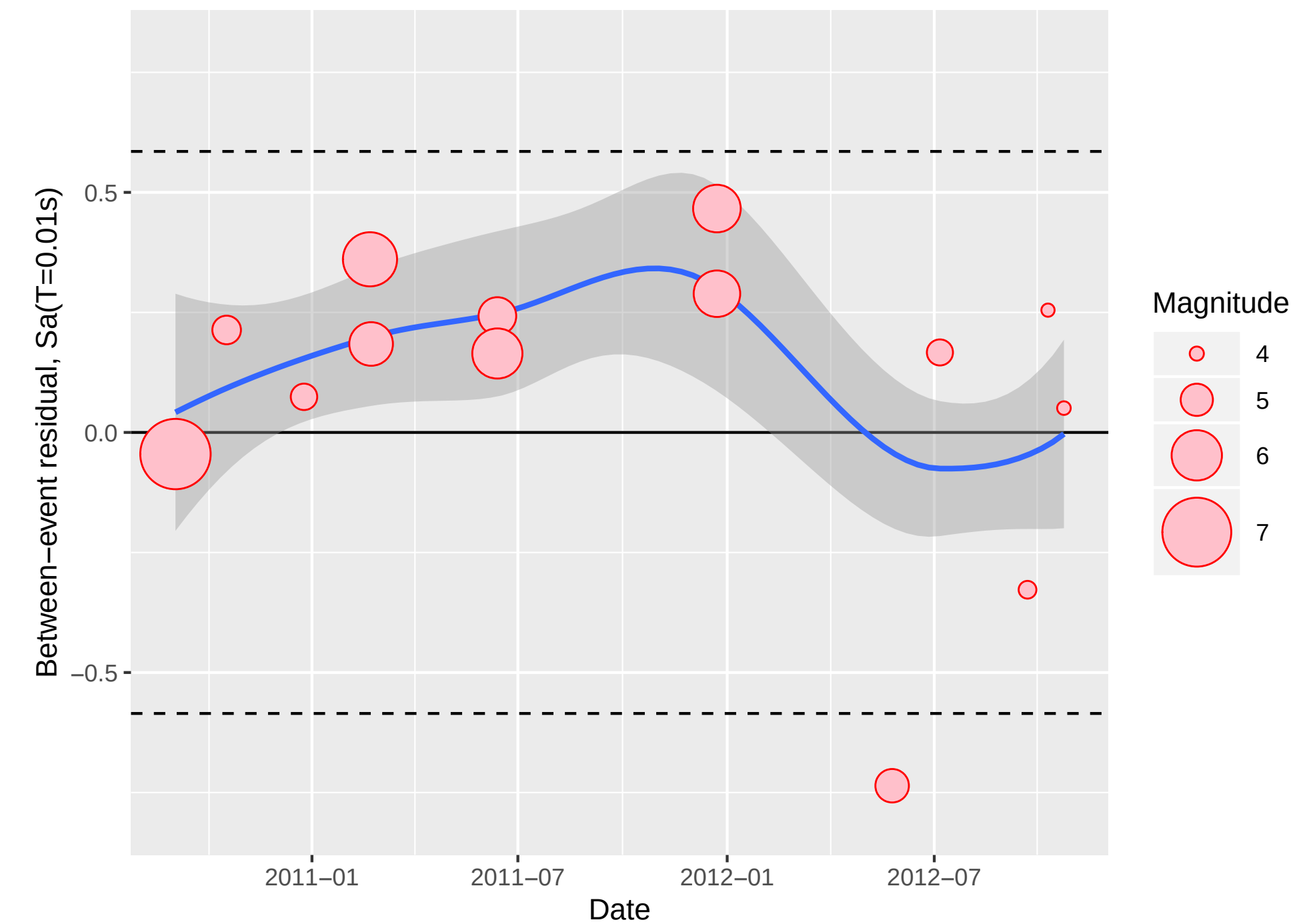
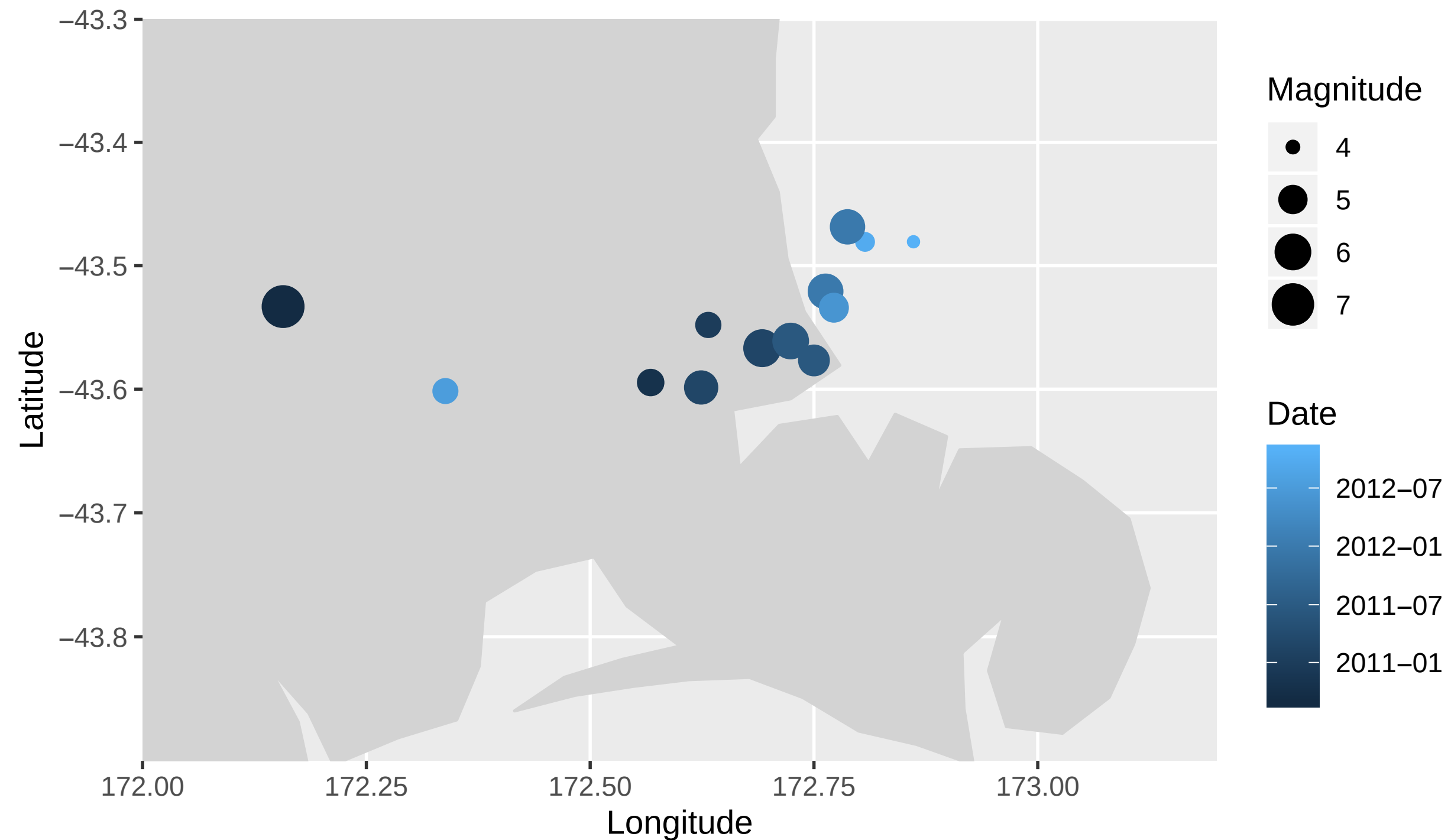
Earthquake sequences

- Identify largest clusters according to the Gardner & Knopoff declustering algorithm
- Look at the between-event residuals (event random effects) for the events within these clusters
- Compare the variation of these random effects in time
- Compare the variability of random effects within cluster to that obtained for the entire NZ database



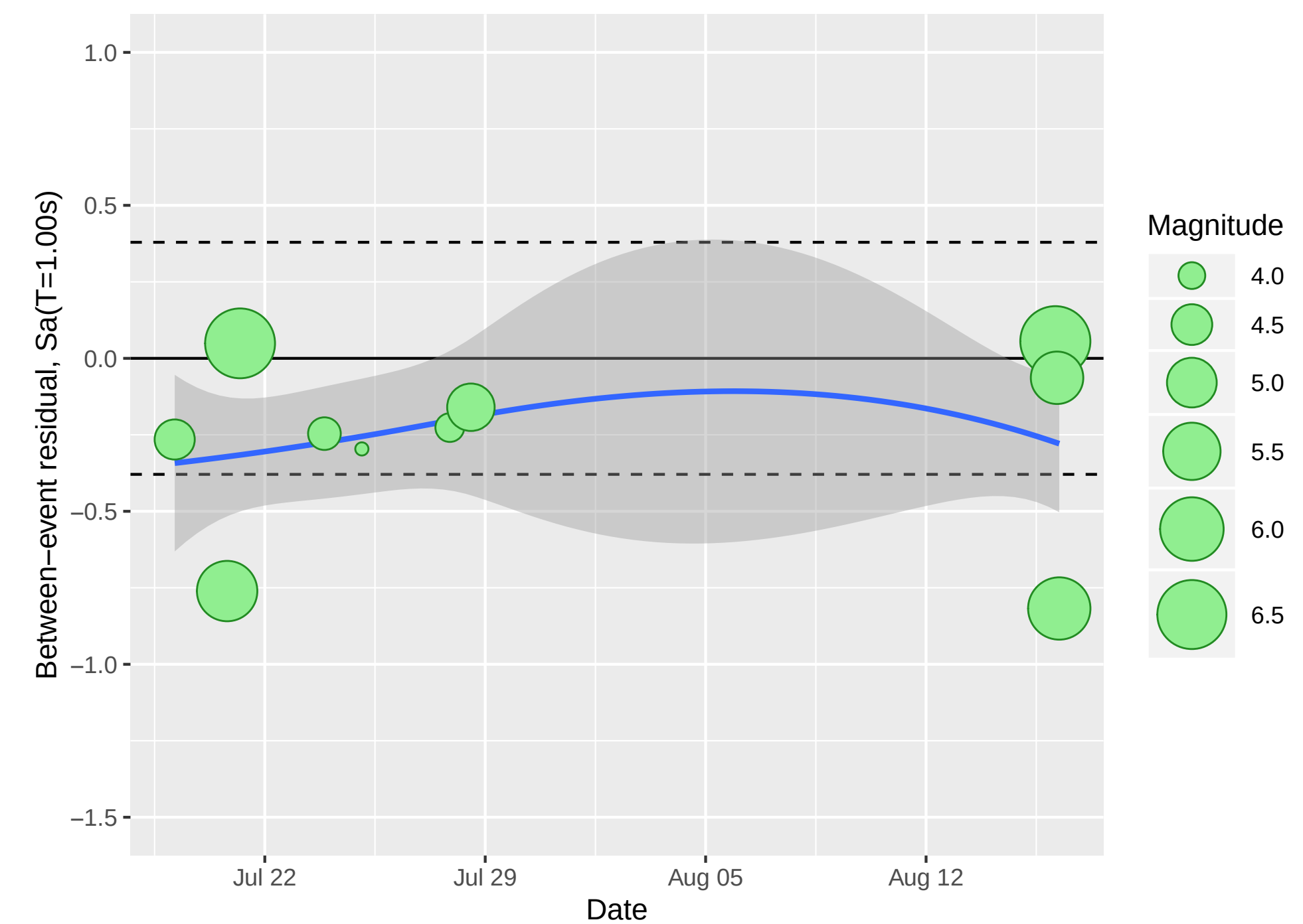
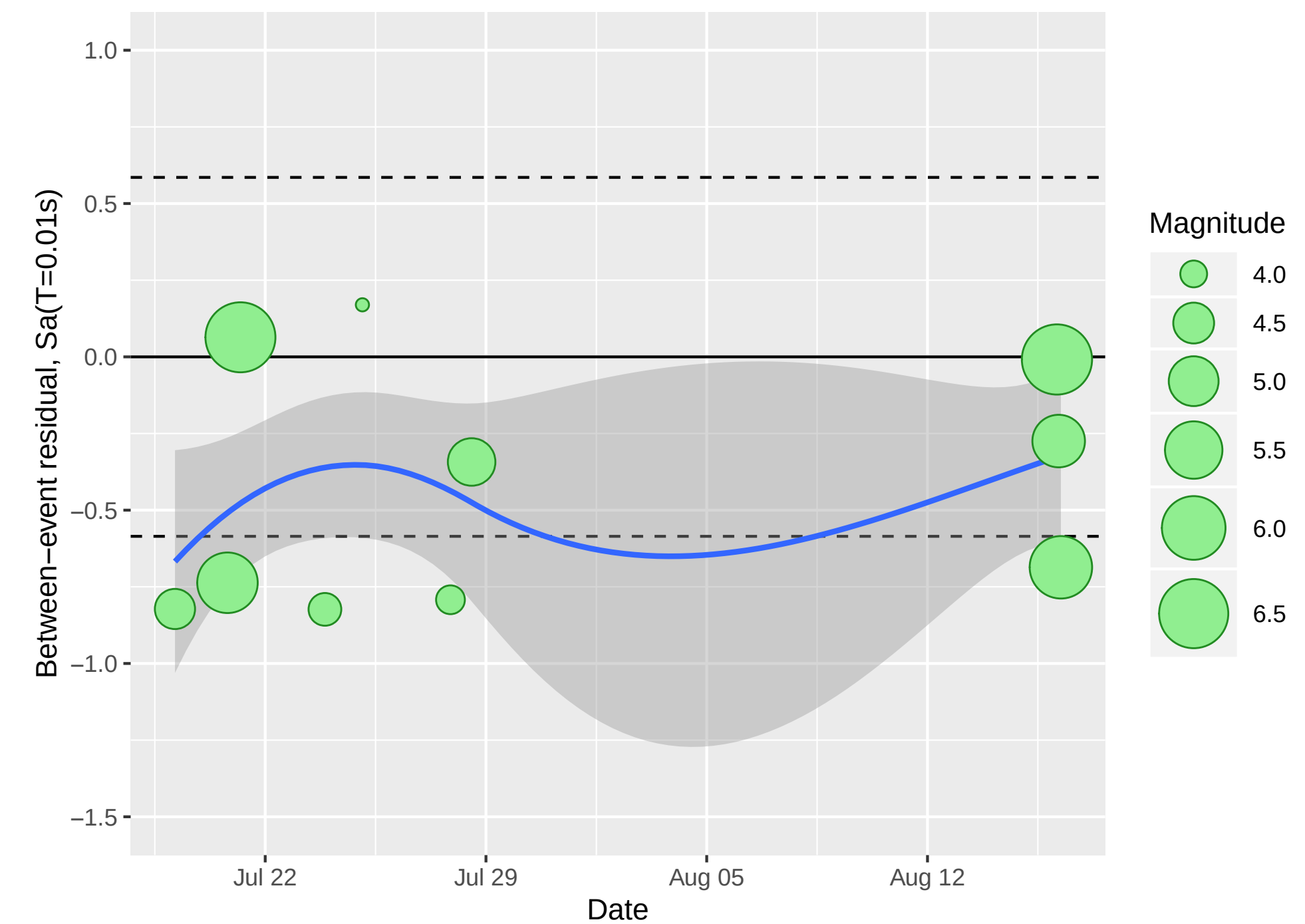
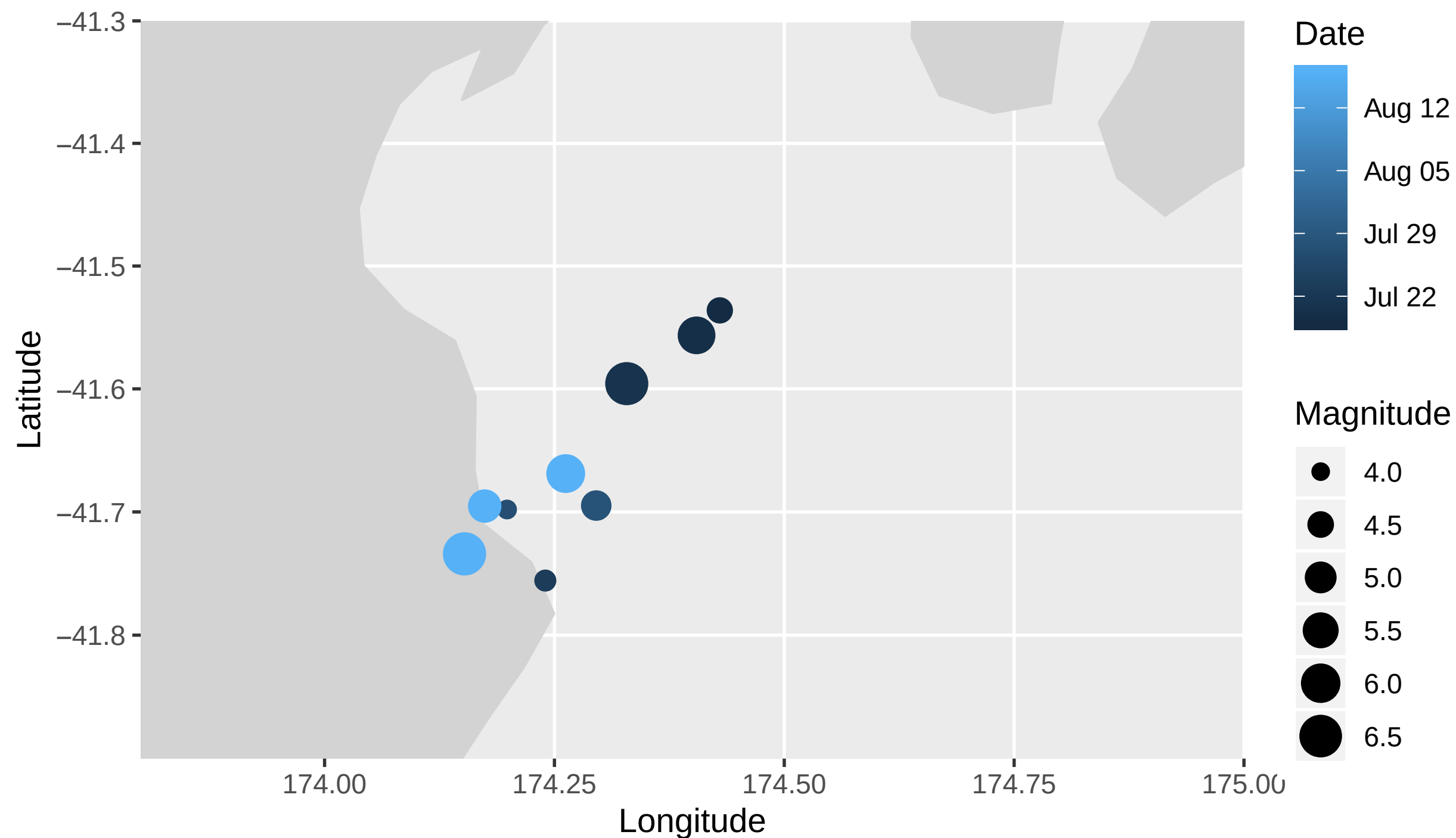
Canterbury sequence 2010-2011

- Particularly for the first year we have within-cluster variability that is significantly lower than ergodic variability
- We can progressively update the estimate of δ_C as each new event arrives
- Higher-than average event terms (higher risk), but lower between-event variability (lower risk)



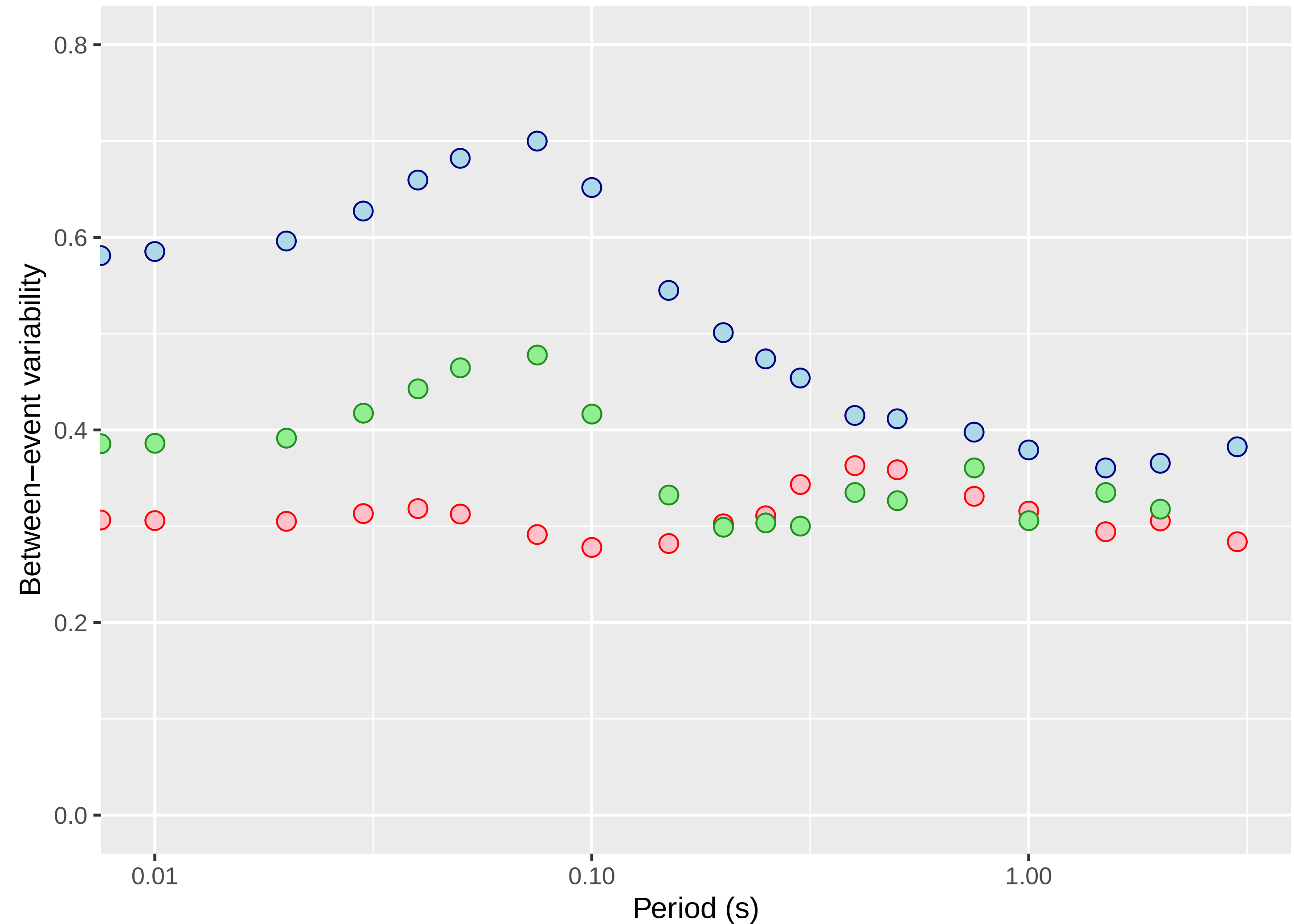
Marlborough sequence 2013

- More of a swarm than a typical aftershock sequence, but still a clear cluster offset, and reduced variability compared to the ergodic model
- Lower than average event terms (lower risk), lower between event variability (lower risk)



Source variability within sequences

- When evaluated across all response periods, the within-cluster variability is clearly lower than the general between-event variability from the entire NZ database
- Two factors at play here, the location specific effect, and the cluster-specific effect



Conclusions

- ⦿ Ground-motion models and correlation models used in risk analyses almost always make use of the ergodic assumption
- ⦿ This leads to inflated estimates of the variance (higher risk) and potentially inflated estimates of correlations among intensity measures
- ⦿ In locations like Istanbul it should be possible to reduce the degree of aleatory variability to reflect the quite specific dominant source (lower between-event variability), and to capitalise upon the relatively good characterisation of the local soil deposits (lower within-event variability, and more accurate spatial correlation models)
- ⦿ For post-mainshock risk analyses, information from the sequence as it develops can be used to refine risk estimates